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Key Points:

- A new wave breaking model suitable for coupled models provides a framework to explicitly account for wave breaking related air-sea fluxes
- The spectral wave breaking crest distributions model is consistent with field observations over a wide range of conditions
- Wave breaking variability due to wave-current interactions is dominated by submesoscale current variability

Supporting Information:

Supporting Information S1

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Distribution of Surface Wave Breaking Fronts

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Abstract This study describes a model of Phillips' $\Lambda(c)$ distribution, which is the expected length of breaking fronts (per unit surface area) moving with velocity c to c + dc, providing a framework for coupled atmosphere-wave-ocean models to explicitly account for wave breaking related air-sea fluxes. The model of Λ depends on the spectral saturation, based on Gaussian statistics of the lengths of crest exceeding wave slope criteria, and long wave-short wave modulation. A wave breaking dissipation function based on Λ was implemented in the model WaveWatchIII. The wave solutions are consistent with the observations, including several metrics of the spectrum and $\Lambda(c)$ distributions. The whitecap coverage derived from Λ reproduces recent parameterizations saturating at high winds. The wave breaking variability due to wave-current interaction is significant at submesoscales (order 1 km or smaller). The wave breaking model can be further developed to model gas transfer coefficients and aerosol production.

Plain Language Summary This study introduces a new model of wave breaking that allows coupled atmosphere-wave-ocean models to explicitly account for wave breaking related air-sea exchanges, as well as direct applications for climate, weather, remote sensing, and biogeochemical studies. The model is based on a physical framework that allows explicit computations of wave breaking related air-sea fluxes including momentum, energy, gases, and aerosols across the air-sea interface. The energy dissipation from the new breaking model was implemented in a spectral wave model. The resulting wave model solutions accurately reproduce the main features of empirical wave energy spectra. The wave breaking distributions, and whitecap coverage from the model are consistent with field observations over a wide range of conditions. The results show that wave breaking variability is significantly modulated due to current variations at small scales.

1. Introduction

Surface wave breaking modulates the fluxes of energy, momentum, mass, and gases across the air-sea interface (Melville, 1996; Perlin et al., 2013). Breaking waves drive upper ocean currents and mixing (Phillips, 1977), affect aerosol production, and enhance gas exchange and heat fluxes across the air-sea interface (Farmer et al., 1993; Thorpe, 1982), all of which have implications for weather and climate change predictions (Loewen, 2002). Air-sea fluxes have been traditionally parameterized with crude models based on wind speed. Phillips (1985) introduced a rational framework to describe the wave breaking kinematics and dynamics through a distribution $\Lambda(c)$, which is the expected length of breaking crests moving with velocity *c* to *dc* per unit surface area, with corresponding azimuth-integrated distribution $\Lambda(c) = \int \Lambda(c)cd\theta$. The moments of $\Lambda(c)$ are related to the following physical parameters:

Total length of breaking fronts per unit surface area:
$$L = \int \Lambda(\mathbf{c}) d\mathbf{c}$$
 (1)

Fraction of total surface area turned over per unit time: $R = \int c \Lambda(c) dc$ (2)

Fractional whitecap coverage:
$$W \propto \int c^2 \Lambda(\mathbf{c}) d\mathbf{c}$$
 (3)

Rate of air entrainment per unit surface area:
$$V_a \propto \int c^3 \Lambda(\mathbf{c}) d\mathbf{c}$$
 (4)

Momentum flux per unit surface area:
$$M \propto \int c^4 \Lambda(\mathbf{c}) d\mathbf{c}$$
 (5)

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Energy dissipation per unit surface area:
$$E \propto \int c^5 \Lambda(\mathbf{c}) d\mathbf{c}$$
, (6)

for details the reader may refer to Phillips (1985), Kleiss and Melville (2010), and Deike et al. (2017).

Several studies have measured Λ using visible and/or infrared imaging techniques under various environmental conditions (Gemmrich et al., 2008; Kleiss & Melville, 2010; Thomson et al., 2009; Zappa et al., 2012; Schwendeman et al., 2014). The High-Resolution Air-Sea Interaction Experiment provided measurements of $\Lambda(c)$ from visible imagery over areas with strong wave-current interactions (Romero et al., 2017), as well as coincident measurements of $\Lambda(c)$ from visible and infrared imagery over a wide range of wave ages (Sutherland & Melville, 2013, hereafter referred to as SM13). The infrared imagery of SM13 allowed quantification of wave breaking statistics for both waves that entrain air and microbreakers. One of the main findings by SM13 was an empirical scaling of $\Lambda(c)$ approximately collapsing the data along a line proportional to c^{-6} .

There are, however, several shortcomings related to Phillips' framework as discussed by Zappa et al. (2016) and Banner et al. (2014). For example, it is not immediately obvious how to relate the speed of a breaking front to a particular scale. Although laboratory measurements suggest it is proportional to the phase speed (Banner & Peirson, 2007; Rapp & Melville, 1990; Stansell & MacFarlane, 2002), the speed of a breaker slows down during the breaking process. Alternatively, other approaches have been proposed directly relating the energy dissipation to the bubble plume volume or the whitecap coverage (Callaghan, 2018; Hwang & Sletten, 2008).

Despite the controversy surrounding Phillips' framework, the findings by SM13 provide a benchmark for the development of a new model of $\Lambda(c)$ suitable for spectral wave models, and in particular for coupled atmosphere-wave-ocean models explicitly accounting for air-sea exchanges due to surface wave breaking. This study describes a new model of Λ , including its implementation and validation within the spectral model WaveWatch III (The WAVEWATCH III Development Group [WW3DG], 2016), here defined as WW3. The wave breaking model builds from the work by Romero and Melville (2011) on the length of crests exceeding wave slope thresholds. The model of $\Lambda(c)$ and its implementation in WW3 are described in section 2. The model results are presented in section 3, which are discussed and summarized in sections 4 and 5, respectively.

2. Methods

2.1. Wave Breaking Model

The directional wavenumber spectrum F(k) of surface waves is defined such that

$$\langle \eta^2 \rangle = \int F(\mathbf{k}) \mathrm{d}\mathbf{k},$$
 (7)

with $\langle \eta^2 \rangle$ corresponding to the variance of the surface elevation. The azimuth-integrated or omnidirectional spectrum is given by

$$\phi(k) = \int F(k,\theta)k\,\mathrm{d}\theta. \tag{8}$$

The spectral evolution of the directional wave spectrum F(k) in deep water can be described through the wave action conservation equation

$$\frac{\partial N(\boldsymbol{k})}{\partial t} + (\boldsymbol{c}_{g} + \boldsymbol{u}) \cdot \frac{\partial N(\boldsymbol{k})}{\partial \boldsymbol{x}} - \boldsymbol{k} \cdot \frac{\partial \boldsymbol{u}}{\partial \boldsymbol{x}} \frac{\partial N}{\partial \boldsymbol{k}} = \frac{S_{\text{in}} + S_{\text{nl}} + S_{\text{ds}}}{\sigma}, \tag{9}$$

where $N(k) = F(k)/\sigma(k)$ is the wave action, $\sigma(k) = (gk)^{1/2}$ is the frequency according to the linear dispersion relationship, *u* is the surface current, and *S* is the sum of the energy sources (Hasselmann et al., 1973). The dominant energy source terms are composed of wind input S_{in} , nonlinear four-wave resonant fluxes S_{nl} , dissipation due to wave breaking S_{ds} .

In this study, the wind input S_{in} is modeled according to Ardhuin et al. (2010) based on the quasilinear theory by Janssen (1991), including short-wave sheltering due to longer waves (Chen & Belcher, 2000) and swell dissipation for waves traveling faster than the wind, using the default tuning parameters of the ST4 package with a maximum roughness length Z0MAX= 8×10^{-4} . Thus, limiting the drag coefficient to 1.85×10^{-3} (Sullivan et al., 2012). The nonlinear energy fluxes are computed with the standard Direct Interaction Approximation (DIA; Hasselmann & Hasselmann, 1985) and the Åexact" Webb-Resio-Tracy method (WRT; Tracy & Resio, 1982, van Vledder, 2006).

The spectral dissipation S_{ds} is computed through $\Lambda(k)$, which is modeled on the assumption that it is proportional to the statistics of crest lengths exceeding a wave slope criterion. Following the work by Longuet-Higgins (1957), Romero and Melville (2011) derived an analytical model describing the statistical distribution of the length of crests per unit surface area and unit bandwidth for waves exceeding wave slope criterion s_0 according to

$$I_{s_o} = \frac{1}{3} \frac{s_o}{2m_{00}^{1/2}\bar{k}} \exp\left(-\frac{s_o^2}{2m_{00}\bar{k}^2}\right),\tag{10}$$

where $m_{00} = \langle \eta^2 \rangle$ and \bar{k} is the mean wavenumber. Romero and Melville (2011) showed good agreement between equation (10) and band-passed airborne lidar wave observations (Romero & Melville, 2010a, hereafter referred to as RM10). Assuming that wave groups are self-similar and a functional form as in equation (10) with respect to the spectral saturation $B(k) = F(k)k^4$, $\Lambda(k)$ is modeled according to

$$\Delta(\mathbf{k}) = \frac{l}{k} \exp\left(-\frac{B_{\rm br}}{B(\mathbf{k})}\right) M_L(\mathbf{k}) M_W(k),\tag{11}$$

where B_{br} is a characteristic saturation representative of the breaking waves, l is a dimensionless function, $M_L(k)$ is a function accounting for the breaking modulation by the longer waves, and $M_W(k)$ is an amplification for the short waves to balance the wind input. The spectral saturation is a convenient parameter to characterize the wave slope locally within a broadband spectrum assuming self-similarity and has been shown to correlate with breaking waves in the field (Banner et al., 2002) and used extensively to model wave breaking (Alves & Banner, 2003; Ardhuin et al., 2010; Banner & Morison, 2010; Donelan, 2001).

Donelan (2001) proposed a nonlinear spectral dissipation function due to wave breaking that accounts for the straining of the short waves by the longer waves in terms of the cumulative mean squared slope cmss(k).

Following Peureux et al. (2018) and Guimaraes (2018), the wave breaking modulation due to longer waves is modeled according to

$$M_L(\mathbf{k}) = \left(1 + 400\sqrt{\mathrm{cmss}(k)}\mathrm{cos}^2(\theta - \theta_w)\right)^{3/2},\tag{12}$$

where the cumulative mean squared slope

$$\operatorname{cmss}(k) = \int_0^k F(k,\theta) k^2 k \mathrm{d}k \mathrm{d}\theta, \qquad (13)$$

and θ_w is the spectrally weighted mean wave direction defined as

$$\theta_{w} = \tan\left(\frac{\int F(\mathbf{k})\sin\theta\,\mathrm{d}\mathbf{k}}{\int F(\mathbf{k})\cos\theta\,\mathrm{d}\mathbf{k}}\right).$$
(14)

The factor of 400 and the power of 3/2 in equation (12) were found to give the best performance against the measurements by RM10 with respect to the bimodal directional distribution at wavenumbers larger than the spectral peak.

The wind modulation function is modeled linearly according to

$$M_W(k) = (1 + D\max[1, \frac{k}{k_o}])/(1 + D)$$
(15)

with $k_o(u_*) = g(\frac{3}{28u_*})^2$. Therefore, $M_W(k)$ is only activated for $k > k_o$ balancing the wind input while maintaining a saturation level at large wavenumbers consistent with the field observations (i.e., RM10; Lenain & Melville, 2017). The dimensionless factor D = 0.9 for the DIA and D = 2 for WRT computations. The larger value of D and a wider range of resolved wavenumbers are needed for WRT computations to minimize the variability of the spectral saturation at large wavenumbers.

By definition, the total length of breaking crests per unit surface area

$$L = \int \Lambda(\boldsymbol{k}) d\boldsymbol{k}$$
(16)



and

$$\Lambda(\boldsymbol{c})d\boldsymbol{c} = \Lambda(\boldsymbol{k})d\boldsymbol{k}.$$
(17)

Following Phillips (1985), an element of area dk corresponds to an area element dc as given by

$$\mathbf{d}\boldsymbol{k} = k\,\mathbf{d}k\,\mathbf{d}\theta\tag{18}$$

$$= -\frac{\partial k}{\partial c} \frac{k}{c} c dc d\theta, \tag{19}$$

which according to the linear dispersion relationship $c = (g/k)^{1/2}$ becomes

$$\mathbf{d}\boldsymbol{k} = -\frac{2g^2}{c^6}c\,\mathbf{d}c\,d\theta\tag{20}$$

$$= -\frac{2g^2}{c^6} \mathbf{d}\boldsymbol{c}$$
(21)

and

 $\Lambda(\mathbf{c}) = -\frac{2g^2}{c^6} \Lambda(\mathbf{k}) \bigg|_{k=gc^{-2}},$ (22)

which integrated in azimuth becomes

$$\Lambda(c) = -\frac{2g}{c^3}\Lambda(k)\Big|_{k=gc^{-2}}.$$
(23)

Laboratory measurements have shown that the speed of breaking crests (c_{br}) is linearly related to the phase speed through a factor α , such that $c_{br} = \alpha c$, with α varying between 0.7 and 0.95 (Banner & Peirson, 2007; Rapp & Melville, 1990; Stansell & MacFarlane, 2002). For consistency with SM13, here it is assumed that $\alpha = 1$.

Duncan (1981, 1983) introduced a scaling of the energy dissipation rate per unit length of breaking crest according to

$$\epsilon_l = \frac{b\rho_w c^5}{g},\tag{24}$$

where *b* is a dimensionless parameter representing the strength of breaking. Laboratory measurements have shown that *b* depends on the wave steepness and the rate of energy convergence of wave groups varying between 1×10^{-4} and 1×10^{-1} (Banner & Peirson, 2007; Drazen et al., 2008; Melville, 1994). Phillips (1985) generalized equation (24) for a broadband spectrum according to

$$\rho_w g S_{\rm ds}(\boldsymbol{c}) d\boldsymbol{c} = \frac{\rho_w}{g} b \Lambda(\boldsymbol{c}) c^5 d\boldsymbol{c}, \qquad (25)$$

such that

$$S_{\rm ds}(\boldsymbol{c}) = \frac{b}{g^2} \Lambda(\boldsymbol{c}) c^5 \text{ or } S_{\rm ds}(\boldsymbol{k}) = \frac{b}{g^2} \Lambda(\boldsymbol{k}) c^5.$$
(26)

Subsequently, Romero et al. (2012) combined a semiempirical quantification of the spectral energy dissipation with measurements of $\Lambda(c)$ by Kleiss and Melville (2010) to show that the strength of breaking *b* is not a constant across the spectrum. They proposed the model

$$b(k) = A(B(k)^{1/2} - B_T^{1/2})^{5/2},$$
(27)

where $B(k) = \phi(k)k^3$ is the azimuth-integrated saturation, A is a dimensionless factor of order unity, and B_T is a breaking saturation threshold (Banner et al., 2002), and the power of 5/2 is based on the inertial scaling by Drazen et al. (2008). Here I use A = 3.8 and $B_T = 1.1 \times 10^{-3}$, which are within the range of values obtained by Romero et al. (2012).

The spectral dissipation due wave breaking $S_{ds}(k)$ is modeled combining equations (11)–(15), (26), and (27). The dissipation written solely in terms of either wavenumber or frequency is included for clarity in the

supporting information. The constants l and B_{br} were tuned under idealized duration-limited runs such that the significant wave height (H_s) and peak wave period (T_p) matched Ardhuin et al. (2010; ST4), and the equilibrium range level matched Resio et al. (2004). The resulting coefficients are $B_{br} = 5 \times 10^{-3}$ and $l = 3.5 \times 10^{-5}$ for solutions forced with either DIA or WRT. The wave breaking saturation parameter B_{br} controls the shape of the spectrum at the peak and/or intermediate wavenumbers. As B_{br} increases, the spectral peakedness and the level of the equilibrium range decrease and vice versa.

After implementation of the $\Lambda(k)$ -based dissipation, two additional output parameters were implemented in WW3: the whitecap coverage and air entraining rate. Following Kleiss and Melville (2010), the whitecap coverage is modeled according to

$$W = \frac{2\pi}{g} \gamma \int c^2 \Lambda(\boldsymbol{c}) \mathrm{d}\boldsymbol{c}, \qquad (28)$$

where γ is a dimensionless factor representing the duration of breaking relative to the wave period $(2\pi c/g)$. Here γ is set to 0.56 and the lower limit of integration c = 2 m/s (Deike et al., 2017). Deike et al. (2017) developed a model for the rate of entrained air volume per unit area of ocean surface (V_a) based on direct numerical simulations. The model was further developed for field conditions in terms of $\Lambda(c)$ according to

$$V_a = \frac{\chi A}{g} \int (B^{1/2}(k) - B_T^{1/2})^{3/2} c^3 \Lambda(\mathbf{c}) d\mathbf{c},$$
(29)

where A is the proportionality factor of the strength of breaking (27) and χ is a constant representing the ratio between the work done by buoyancy forces and mechanical dissipation, here set to $\chi = 0.2$. Again, the lower limit of integral (29) is set to 2 m/s. Deike and Melville (2018) subsequently extended the model of air entrainment rate to model the bubble-mediated gas transfer coefficient of CO₂, which can be easily adapted in WW3.

2.2. Duration-Limited Experiments

Following Romero (2008), duration-limited experiments were conducted with stationary winds for model tuning and validation. The model runs consist of the source term integration test case of WW3 (ww3_ts1). The spectrum was discretized with a resolution $\Delta \theta = 10^{\circ}$ in azimuth and $\Delta k/k = 0.14$ in wavenumber. The lowest wavenumber resolved is 0.0016 rad/m and the upper is 4 and 10 rad/m for the DIA and WRT, respectively. A larger upper value is needed with "exact" computations of the nonlinear energy fluxes to minimize the variability of the degree of saturation at large wavenumbers. The global time step was set to 300 s, and the minimum dynamical the source term integration step was set to 15 s. All model computations in this study allow the tail of the spectrum to evolve freely without a prognostic tail (Liu et al., 2019).

The model was integrated for 12 days starting with "calm" conditions ($H_s = 0$) with the linear wind input source term enabled (Cavaleri & Malanotte-Rizzoli, 1981). Duration-limited solutions were generated for wind speeds between 4 and 35 m/s, saving output every 20 min. The model was tuned against ST4 solutions of H_s and T_p with normalized root-mean-square errors (NRMSE; equation (26) of Ardhuin et al., 2010) of less than 5% for winds speeds between 4 and 35 m/s.

2.3. Regional Model Configuration

There is a growing interest within the community to better understand the modulation of the wave field, including wave breaking, due to wave-current interactions (Ardhuin et al., 2017; Romero et al., 2017; Zippel & Thomson, 2017). Here, the wave breaking variability due to wave-current interactions is investigated with a regional model configuration forced with surface currents from a high-frequency radar array with a horizontal resolution of 2 km (Kaplan & Largier, 2006; Kaplan et al., 2005). The model solutions are validated against lidar wave observations collected from an aircraft (Romero et al., 2017). The grid has 81×64 points in space with a resolution of 0.01° (1 km). The bathymetry consists of the 1 km product SRTM30_PLUS from Becker et al. (2009). The time steps were set to 20 s with the minimum dynamical source term integration of 5 s. The spectral grid has an azimuthal resolution of 10° and $\Delta k/k = 0.2$ with wavenumbers between 0.0016 and 4 rad/m. The initial and boundary conditions were generated using the measured directional wavenumber spectra (Romero et al., 2017) extrapolated from 0.4 to 4 rad/m following Romero et al. (2012). Boundary conditions were kept constant but the forcing winds and currents varied in time. Time-varying forcing surface winds were provided homogeneously using the field measurements from FLIP (Grare et al., 2013) with a temporal resolution of 30 min. For comparison, an additional model run was forced with relative winds (i.e., $U_{10} - u(x)$, where u(x) corresponds to the surface current).





Figure 1. Evolving omnidirectional spectra (a) and corresponding compensated (saturation) spectra ϕk^3 (b) color coded by wave age, with reference power laws of $k^{-5/2}$ and k^{-3} shown with black solid and dashed lines, respectively. Toba's coefficient β is plotted against the wave age c_p/u_* in (c) and compared against Resio et al. (2004) and RM10. (d, e) Directional spreading $\sigma_{\theta}(k/k_p)$ from DIA and WRT solutions, respectively. (f) The directional spreading data from RM10 (solid lines), with dashed lines colored lines indicating the error magnitude. The black dashed lines in (d)–(f) correspond to the directional spreading from lidar observations by Hwang et al. (2000). DIA = Direct Interaction Approximation; WRT = Webb-Resio-Tracy method.

3. Results

3.1. Wave Spectrum

In this section, the wave model performance is analyzed with an idealized solution forced with 20-m/s winds. Omnidirectional spectra at various wave ages c_p/u_* are shown in Figure 1a, comparing DIA and WRT computations. The solutions show the typical differences with narrower spectra for the exact nonlinear energy fluxes (Hasselmann & Hasselmann, 1985). The shape of the omnidirectional spectra above the spectral peak approximately follows two power law regimes: an equilibrium range $\phi \sim k^{-5/2}$ followed by a saturation range $\phi \sim k^{-3}$, in agreement with the measurements (Lenain & Melville, 2017; Long & Resio, 2007; Romero & Melville, 2010a). This is better shown with $B(k) = \phi(k)k^3$ in Figure 1b. The energy level within the equilibrium range is slightly larger for the DIA solutions. Following Toba (1973), the equilibrium range can be scaled according to

$$\phi(k) = \frac{\beta}{2} u_* g^{-1/2} k^{-5/2},\tag{30}$$

where β is a dimensionless factor with weak dependence on the wave age c_p/u_* (Resio et al., 2004, RM10). Following Liu et al. (2019), β is calculated from model solutions for 2.25 $k_p < k < 0.35$ rad/m, with 0.35 rad/m corresponding to the upper limit before the noise floor of the data collected by RM10. The mean compensated spectrum is calculated according to

$$\langle \phi k^{5/2} \rangle = \frac{1}{0.35 - 2.25k_p} \int_{2.25k_p}^{0.35} \phi(k) k^{5/2} \mathrm{d}k,$$
 (31)

and

$$\beta = \frac{2g^{1/2}}{u_*} \langle \phi k^{5/2} \rangle.$$
(32)

Figure 1c shows β versus wave age from DIA and WRT solutions compared against the measurements by Resio et al. (2004) and RM10. The model solutions are in good agreement against Resio et al. (2004), roughly 20% larger than RM10. At large wavenumbers, the saturation level approximately reaches that reported by Lenain and Melville (2017) of 7×10^{-3} and RM10 of $7.3 \pm 1.0 \times 10^{-3}$.





Figure 2. (a) $\Lambda(c)$ model solutions compared to SM13 (b), color coded by wind speed, with corresponding nondimensionalized distributions in (c) and (d), respectively. The black straight lines show reference power laws of c^{-6} . The modeled nondimensionalized $\Lambda(c)$ distributions are in good agreement with the observations (gray colors), collapsing along the 0.05 \hat{c}^{-6} line, within the scatter of the data.

The wave solutions are further characterized with respect the directional spreading calculated according to

$$\sigma_{\theta} = \frac{\int_{-\pi/2}^{\pi/2} F(k,\theta) |\theta| d\theta}{\int_{-\pi/2}^{\pi/2} F(k,\theta) d\theta},$$
(33)

limiting the integral to $\pm \pi/2$ for consistency with the RM10 lidar data, which has a 180° ambiguity. The spreading $\sigma_{\theta}(k/k_p)$ is shown in Figures 1d and 1e for the DIA and WRT solutions, respectively, in good agreement against RM10 (shown in Figure 1f). Overall, the WRT solution gives better spreading against RM10, approaching the spreading reported by Hwang et al. (2000) for fully developed seas. This is a substantial improvement compared to other WRT solutions which tend to give much narrower spectra (Liu et al., 2019; Romero & Melville, 2010b).

3.2. Wave Breaking Distributions

In this section, the $\Lambda(c)$ distributions from duration-limited solutions are compared against SM13 in Figures 2a and 2b. The comparison is not one-to-one because the observed Λ 's include very old waves reaching values of $c_p/u_* = 100+$, which suggests a mixture of wind-sea and swell. Nevertheless, the agreement between the model and observations is reasonable. The model solutions are further analyzed with the empirical scaling proposed by SM13, specifically $\Lambda' = \Lambda(c)c_p^2g^{-1}\sqrt{c_p/u_*}$ and $c' = c/\sqrt{gH_s}(gH_s/c_p^2)^{0.1}$. Figures 2c and 2d shows both sets of $\Lambda(c)$ scaled accordingly, collapsing along $0.05 c'^{-6}$.

3.3. Whitecap Coverage and Air Entrainment

The computed whitecap coverage W over a wide range of winds speeds (4 m/s $\leq U_{10} < 35$ m/s) from duration-limited solutions is shown in Figure 3a. The results compare well with the parameterizations by Schwendeman and Thomson (2015), Callaghan et al. (2008), and Brumer et al. (2017). The model solutions exhibit a saturating trend for large wind speeds consistent with the observations, within the scatter of



Figure 3. (a) Whitecap coverage solutions as a function of wind speed, color coded by wave age. The green, orange, and gray dashed lines show the parameterizations by Brumer et al. (2017), Callaghan et al. (2008), and Schwendeman and Thomson (2015), respectively. The yellow squares correspond the data by Holthuijsen et al. (2012). (b) Rate of entrained air volume per unit area of ocean surface (V_a , small circles) normalized by the peak phase speed (c_p) as a function of friction velocity normalized by the ballistic velocity (c_p/H_s)^{1/2}. Data are plotted over the figure by (Deike et al., 2017), color coded by the significant slope criteria ($H_sk_p = H_sg/c_p^2$).

the data by Holthuijsen et al. (2012). The saturation of Λ and its moments comes from its dependence on $\exp(-B_{\rm br}/B(\mathbf{k}))$, which tends to saturate as the equilibrium range level increases with increasing wind speed.

The rate of entrained air volume per unit surface area V_a normalized by peak wave speed c_p is plotted in Figure 3b as a function of the friction velocity u_* normalized by the ballistic velocity \sqrt{gHs} for winds between 4 and 16 m/s. The data are plotted over the values obtained by Deike et al. (2017) based on field measurements of $\Lambda(c)$. The model results show good agreement with Deike et al. (2017). The air entrainment rate computations within WW3 can be easily extended to model the bubble-mediated gas transfer of CO₂ based on the work by Deike and Melville (2018).

3.4. Wave-Current Interactions

The wave breaking variability due to wave-current interactions is investigated with the regional model configuration in Northern California as described in section 2.3. Romero et al. (2017) collected novel field observations of directional wavenumber spectra and wave breaking statistics across the edge of an upwelling jet, which overlapped with a narrow area with enhanced wave breaking along a submesoscale front. Here I focus on the wave breaking solutions across the upwelling jet/submesoscale front. A snapshot of the model whitecap coverage is plotted to compare the field observations in Figure 4a. The modeled whitecap coverage is qualitatively consistent along the flight track across the jet/front, with significant differences toward the northeastern part of the sampling region. The location of the submesoscale front is shown with a thick black line at the edge of the jet. The observed whitecap coverage exhibits considerably larger spatial variability than the model solutions likely in part due to statistical sampling and submesoscale current variability not resolved by the HF radar. The model solutions of $\Lambda(c)$ on either side of the submesoscale front along the flight track crossing the front are shown in Figure 4b in qualitative agreement against the field observations for c > 3 m/s. At lower speeds, the observed $\Lambda(c)$ distributions from visible imagery roll-off much faster. The $\Lambda(c)$ distributions are larger on the colder side of the front primarily due to direct forcing and straining of the waves by the currents across the upwelling jet. The effect of relative winds against spatially homogeneous winds is small, which can be seen by comparing the thick and thin red and blue lines in Figure 4b.

4. Discussion

The results demonstrate that the new dissipation model based on Λ gives excellent wave model performance with spectra characterized by two power law regimes (equilibrium and saturation) consistent with empirical data, and directional spreading consistent with that reported by RM10. Moreover, the directional distribution of the spectrum above the spectral peak is bimodal with lobe separations consistent with RM10. The bimodality is mainly due to the modulation transfer function which dissipates more energy in the



Figure 4. (a) Snapshot of modeled whitecap coverage *W* compared to the measurements by Romero et al. (2017; circles). The gray arrows show the HF radar surface currents. The black arrow shows the wind direction and the thick black line indicates the location of the observed submesoscale front at the edge of the upwelling jet. (b) Model and observed $\Lambda(c)$ across the submesoscale front. The red (pink) and blue (light blue) lines correspond to model solutions (observations—R17) on the warm and cold sides of the front, respectively. The model solutions shown with thick lines where forced with homogeneous winds as opposed to those forced with relative winds (RW) shown with thick lines. For reference, see $\Lambda(c)$ sampling flight tracks in Romero et al. (2017, Figure 7e).

direction of the longer dominant waves, which favors the nonlinear energy fluxes (Peureux et al., 2018). This has important implications for directionality of the mean squared slope and the prediction of microseisms (Munk, 2009) and will be explored elsewhere.

An advantage of a dissipation model based on Λ is that it can be directly used to calculate the various wave breaking related parameters and air-sea fluxes, including the effects of wave-current interactions. The test case presented with wave-current interactions based on relatively coarse HF radar currents gave reduced spatial variability of breaking compared to the observations. This is further explored here with a wave model configuration with 100 m resolution, forced with currents from the Regional Ocean Modeling System (Shchepetkin & McWilliams, 2005), and the Weather Research and Forecasting Model. Figure 5a shows a snapshot of simulated whitecap coverage (W) off the central coast of California with substantial variability due to wave-current interactions. Figure 5b shows a zoom over an area with enhanced wave breaking qualitatively similar to the photograph in Romero et al. (2017; Figure 1). The "line" of increased wave breaking overlaps with a submesoscale front with strong horizontal shear, surface convergence, and strong downwelling (Figures 5c and 5d). These effects are consistent with Rascle et al. (2017) who showed that the mean squared slope is enhanced along fronts in conditions with winds blowing obliquely to the front due to increased wave straining downwind by the current gradients. Similarly, the line of enhanced wave breaking in Romero et al. (2017) occurred in conditions with winds obliquely aligned with a submesoscale front. As discussed by Romero et al. (2017) enhanced wave breaking over submesoscale fronts with strong surface convergence and downwelling velocities have potentially important implications for gas transfer. This and a wide range of phenomena due to wave breaking modulated air-sea fluxes including feedbacks can be investigated with a coupled atmospheric-wave-ocean model within the framework of this study.

Finally, as discussed earlier, this study assumes that the observed speeds of the breaking waves (c_{br}) corresponds to the linear phase speed (*c*). But laboratory measurements suggest that $c_{br} = \alpha c$, with α between 0.7 and 0.95. This implies that the observed $\Lambda(c_{br})$ when converted to $\Lambda(c)$ becomes

$$\Lambda(c) = \Lambda(c_{\rm br}) \Big|_{ac} \frac{\partial c_{\rm br}}{\partial c}.$$
(34)

Then assuming that $\Lambda(c_{\rm br}) \sim c_{\rm br}^{-6}$ as approximately supported by the observations, equation (34) would give $\Lambda(c) \sim \alpha^{-5}c^{-6}$. In that case, in order to maintain the same level of dissipation within the model while matching the observed Λ with $\alpha \neq 1$, the parameters *l* and *A* would have to be adjusted accordingly with compensating factors α^{-5} and α^{5} , respectively. This would increase $\Lambda(c)$ by a factor between 6 and 1.3 and reduce *b* by a factor between 0.16 and 0.77 for values of $\alpha = 0.7$ and 0.95, respectively. However, this level of uncertainty is well within the uncertainty of the empirical scaling of Λ' (Figure 2d), and also within the scatter of the observed variability of the strength of breaking from laboratory experiments (Romero et al.,



Figure 5. (a) Simulated whitecap coverage along the central coast of California. (b) Zoom over area with enhanced wave breaking, with corresponding vorticity normalized by the Coriolis parameter (c) and vertical velocity (d). The large arrow in (b)–(d) show the mean wind direction. The arrows in (c) show the surface currents.

2012; Wang & Wijesekera, 2018). It is expected that the uncertainties will become smaller in the future as more measurements become available and new techniques are developed. The interested reader may refer to the supplemental material for additional plots that show the spectral strength of breaking and its effective value against field measurements by Zappa et al. (2016) and references within. Other plots show the azimuth-integrated dissipation $S_{ds}(k)$, and the product $b(c)\Lambda(c)$ comparing favorably with expected power laws within the equilibrium range according to Phillips (1985).

5. Conclusions

This study describes a model of Λ suitable for coupled atmosphere-wave-ocean models. The wave breaking model depends on the spectral saturation and is based on Gaussian statistics of the lengths of crests exceeding wave slope criteria. The breaking modulation due to long wave-short wave interaction is incorporated nonlinearly through the cumulative mean squared slope. Direct modulation by the wind is included at large wavenumbers to match the wind input and ensure a saturation level consistent with the observations. A wave breaking dissipation based on Λ was implemented in WW3 within the source term package by Ardhuin et al. (2010). The wave model performance is excellent, producing spectra with approximately two power law regimes, an equilibrium range ($\phi \sim k^{-5/2}$) followed by a saturation range ($\phi \sim k^{-3}$). The wave model gives directional spreading rates very similar to those reported by RM10, including solutions forced by the exact nonlinear resonant interactions which typically result in much narrower spectra (Romero & Melville, 2010b).

The $\Lambda(c)$ model is consistent with the field observations by SM13. The derived whitecap agrees with recent empirical parameterizations for wind speeds between 4 and 35 m/s, saturating at higher winds. The modeled rate of air entraining agrees with the calculations by Deike et al. (2017) from measurements of $\Lambda(c)$. Wave breaking variability due to wave-current interactions was investigated with currents measured by HF radar. The results are qualitatively consistent with the measurements by Romero et al. (2017), but the Acknowledgments

The model WaveWatchIII (v5.16) with the implementation of the Λ -based dissipation (STX) is available at https://github.com/Leonel-Romero/ WW3-Lambda website. The repository includes scripts to run the idealized cases and the regional configuration with the data used for validation. This work was supported by a grant from ONR (N00014-16-1-2936). I appreciate the encouragement from Ken Melville to pursue this work. I also thank Jim McWilliams and Delphine Hypolite for providing the bathymetry, surface currents from ROMS, and winds from WRF needed to generate Figure 5 in section 4. Realistic model runs were carried out on DoD HPC resources. I am grateful to the Kavli Institute for Theoretical Physics, supported by NSF under Grant No. NSF PHY-1748958, and the program "Planetary Boundary Layers in Atmospheres, Oceans, and Ice on Earth and Moons," where part of this work was completed.

measurements exhibit substantially more variability at small scales, likely due to statistical sampling and submesoscale current variability not resolved by the HF radar.

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