Wave generation by wind

Based on

ON THE GENERATION OF SURFACE WAVES BY SHEAR FLOWS
by John W. Miles (1957)

and

THE INTERACTION OF OCEAN WAVES AND WIND (Chapter 3)
by Peter Janssen (2004)

Action balance equation:

$$\frac{\partial N}{\partial t} + \left(C_g + U\right).\nabla N = S_{in} + S_{nl} + S_{diss}$$

$$E = \rho_w g \langle \eta^2 \rangle = \iint N(\omega, \theta) d\omega d\theta$$

$$\tau = \tau_{tang} + \tau_{form} = \left\langle \frac{\tau_{visc}}{\sqrt{1 + \left(\frac{\partial \eta}{\partial x}\right)^2}} \right\rangle + \left\langle p_s \frac{\partial \eta}{\partial x} \right\rangle$$

$$S_{in}(\omega) = \langle \tau_{visc}.\underbrace{u_s(\omega)}_{\text{Wave-coherent surface velocity}} \rangle + \left\langle p_s \frac{\partial \eta}{\partial x}(\omega) \right\rangle c(\omega) = \left(\underbrace{\tau_{wc}(\omega)}_{\text{Wave-coherent tangential stress}} + \underbrace{\tau_{form}(\omega)}_{\text{Form drag}}\right).c(\omega)$$

THE INTERACTION OF OCEAN WAVES AND WIND by Janssen 2004.
(Chapter 3 - On the generation of ocean waves by wind)

3.0 Introduction

→ Presentation of main theories: Jeffreys (1924), Phillips (1957) and Miles (1957).

→ Presentation of main numerical simulation of the airflow turbulence.

→ Introduction to the quasi-linear theory of wind-wave generation, i.e. feedback of the waves on the mean airflow.

3.1 Linear (quasi-laminar) theory of wind-wave generation from Miles 1957:

→ Equations, linearization, BCs, normalization, critical layer, grow rate formulation

→ Field data showing the importance of the critical layer

3.2 Numerical solution and comparison with observations

3.3 Effects of turbulence:

→ Effects of small-scale turbulence.

→ Rapid distortion of turbulence (Belcher & Hunt).

→ Effect of gustiness.

3.4 Quasi-linear theory of wind-wave generation:

→ Dynamics of the airflow inside the critical layer

→ wave-mean flow interaction. Effect of wave-induced momentum on mean airflow.

3.5 Parametrization of Quasi-linear Theory:

→ parameterization of the source term S_{in}

3.6 Summary of Conclusions

Presentation of main theories:

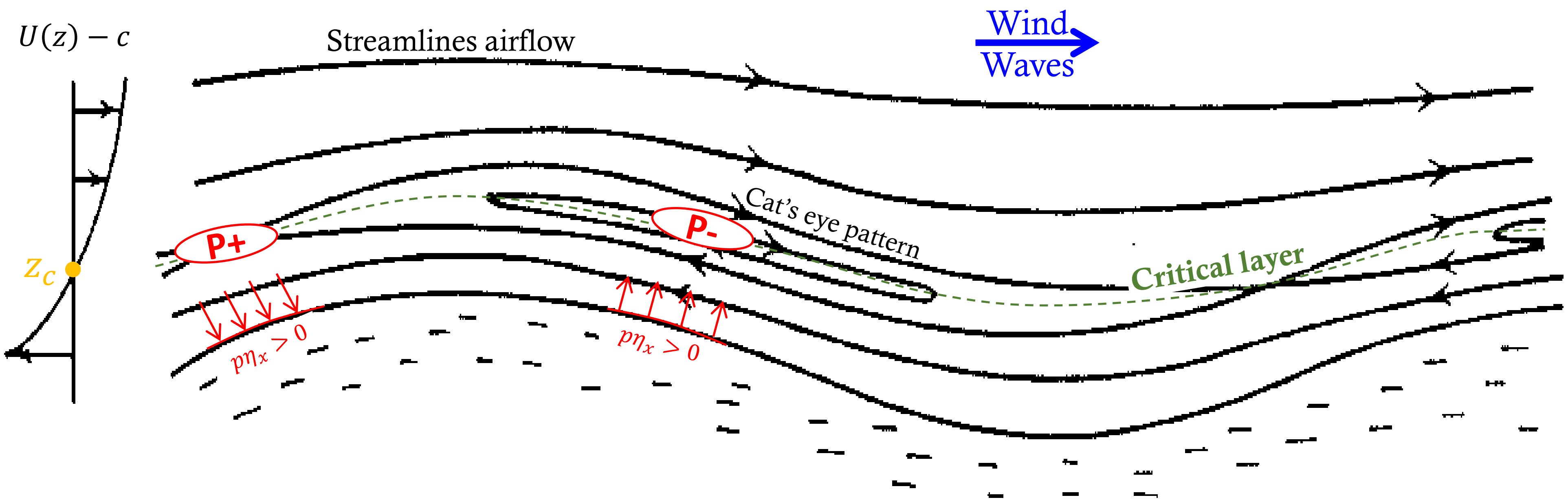
- Jeffreys (1924):
  - Sheltering of the airflow by the waves on their lee side which gives a pressure difference $P(x,t)$ along the wave profile which is responsible for a positive form drag τ_p

    $$P(x,t) = s\rho_a(U-c)^2\frac{\partial\eta}{\partial x}(x,t)$$ where s is the sheltering coefficient.
  - "laboratory measurements on solid waves showed that the pressure difference was much too small to account for the observed growth rates. As a consequence, the sheltering hypothesis was abandoned…"

- Phillips (1957):
  - Resonant forcing of surface waves by turbulent pressure fluctuations.
  - Forcing is maximal if phase speed of surface perturbation matches speed convection of turbulent pressure fluctuations.
  - Linear growth of the wave spectrum with time.
  - This mechanism can explain the first stage of generation of the waves by the wind but not their amplification when the waves amplitude increases.

- Miles (1957):
  - Resonant interaction between wave-induced pressure fluctuations and the free surface waves.
  - Exponential grow of the waves amplitude and energy.
  - Simplified theory: inviscid flow, quasi-laminar approach (i.e. turbulence does not play a role except in maintaining the shear flow), non-linear effects (wave-mean flow interaction) are neglected.

Miles (1957): On the generation of surface waves by shear flows.



Streamlines of the airflow over waves in a frame of reference moving at the phase speed c of the waves. The closed loops are centered at the critical height and shifted downwind of the crest for a growing wave. (From Phillips 1977).

Miles (1957) introduced the concept of the critical layer at height z_c where $U(z_c) = c$.

Miles (1957): On the generation of surface waves by shear flows.

Hypothesis:

- Inviscid fluid.
- Quasi-laminar flow
- 2D flow.
- $\eta(x,t) = ae^{ik(x-ct)}$
- $p(x,t) = (\alpha + i\beta)\rho_a u_*^2 k\eta(x,t)$

$$\rho_a(u_t + Uu_x + wU_z) = -p_x$$
$$\rho_a(w_t + Uw_x) = -p_y$$
$$u_x + w_z = 0$$

Stream function $\psi(x,t)$such that $\psi(x,t) \propto e^{ik(x-ct)}$ and

$$u = -\psi_z \; ; \; w = \psi_x$$

leads to the Rayleigh equation (i.e. inviscid form of the Orr-Sommerfeld equation)

Miles (1957): On the generation of surface waves by shear flows.

Stream function $\psi(x,z,t)$ such that $\psi(x,z,t) \propto \varphi(z).e^{ik(x-ct)}$ and

$$u = -\psi_z \;\; ; \;\; w = \psi_x$$

leads to the Rayleigh equation (i.e. inviscid form of the Orr-Sommerfeld equation)

$$(U-c)(\psi_{zz} - k^2\psi) - U_{zz}\psi = 0,$$

which has a singularity at $U = c$.

Introduction of dimensionless variables:

$$\xi = kz\,;\; \varpi(\xi) = \frac{U(z)-c}{u_*}\,;\phi(\xi) = \frac{\psi(x,z,t)}{u_*.\eta(x,t)}$$

leads to

$$\phi'' - \left[1 + \frac{\varpi''}{\varpi}\right]\phi = 0$$

where all variables depend only on ξ. Obviously there is still the singularity at $U(z) = c$.

Miles (1957): On the generation of surface waves by shear flows.

Boundary conditions: “The boundary conditions to be imposed on ϕ are dictated by the requirements that the interface (originally at $z = z_0$) shall remain a streamline and that the disturbance shall die out at infinity.”

$$\xi = kz\,;\ \varpi(\xi) = \frac{U(z) - c}{u_*}\,;\phi(\xi) = \frac{\psi(x, z, t)}{u_*.\,\eta(x, t)}$$

1st condition:

$$\phi_0 = \phi(\xi_0) = \varpi_0 = \varpi(\xi_0)$$

2nd condition:

$$\phi \rightarrow 0\ as\ \xi \rightarrow \infty$$

Miles (1957): On the generation of surface waves by shear flows.

The final set of equations governing the aerodynamic boundary value problem is:

$$\phi'' - \left[1 + \frac{\varpi''}{\varpi}\right]\phi = 0$$

$$\phi_0 = \varpi_0$$

$$\phi \to 0 \; as \; \xi \to \infty$$

$$\alpha + i\beta = \varpi_0(\phi_0' - \varpi_0')$$

where β characterizes the phase shift between the pressure at the surface and the surface elevation, hence the form drag, hence growth rate of the waves.

$$\beta = -\pi|\phi_c|^2\left(\frac{\varpi_c''}{\varpi_c'}\right)$$

Where the subscript c implies evaluation at $\xi = \xi_c$, i.e. at the critical layer.

Miles (1957): On the generation of surface waves by shear flows.

$$\beta = -\pi|\phi_c|^2 \left(\frac{\varpi_c''}{\varpi_c'}\right)$$

where $|\phi_c|$ depends on the stream function at z_c (i.e. depends on the amplitude of the wave-induced airflow at the height of the critical layer,
and ϖ_c'' depends on the curvature of the mean wind profile at the critical layer.

→ This result implies that*, in the absence of dissipative forces*, a motion of the type $\eta(x,t) = ae^{ik(x-\omega t)}$ will be stable or unstable according as the curvature of the wind profile (U”) at that elevation where the wind speed is equal to the wave speed is positive or negative, respectively.

In Janssen (2004), the normalized variables are different

$$\chi = \frac{w}{w(0)} \ ; W = U(z) - c$$

The resulting growth rate of the waves becomes:

$$\frac{\gamma_a}{\frac{\rho_a}{\rho_w}\omega} = -\frac{\pi}{2k}\frac{W_c''}{W_c'}|\chi_c|$$

Which is also depends on the amplitude of the wave-induced vertical velocity at the height of the critical layer and on the curvature of the mean wind profile at the critical layer.

With this formulation Janssen also shows that the growth rate can be related to the wave-induced momentum flux (or wave-induced stress) $\tau_w = -\overline{\tilde{u}\tilde{w}}$.
Further analysis shows that the wave-induced stress is constant with height except at the critical height where it shows a jump.

For a given wind profile, the boundary value equations can be solved using different techniques (Miles 1957, Conte & Miles 1959, van Duin & Janssen 1992 and Miles 1993). Regardless of the method used to solve this problem there is a fair agreement between the Miles' theory and observations compiled by Plant (1982)

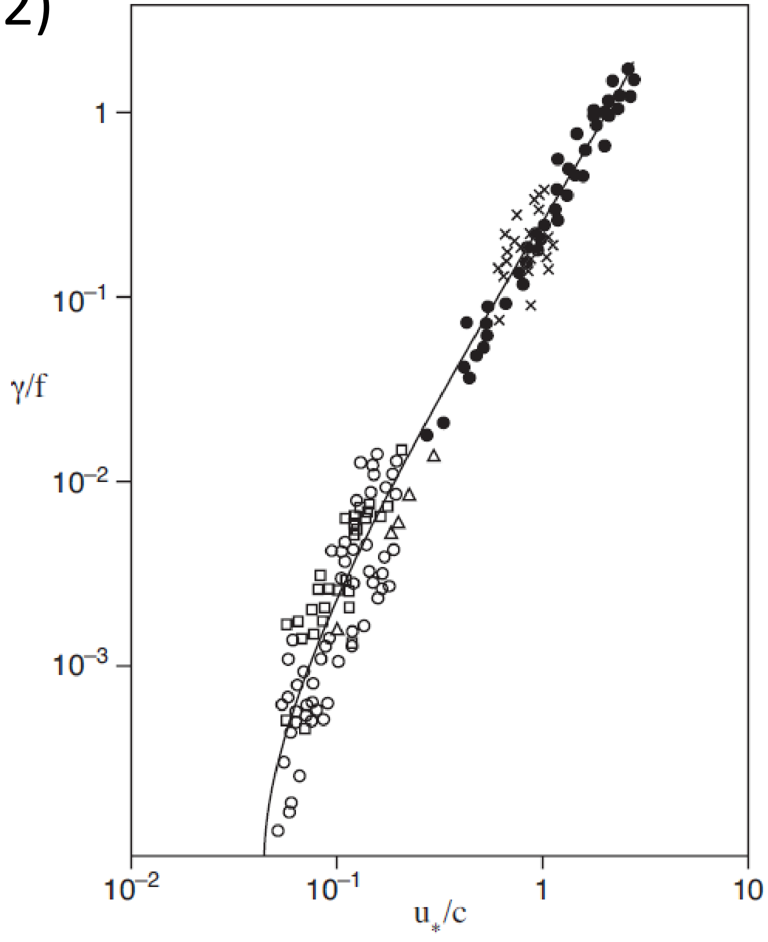
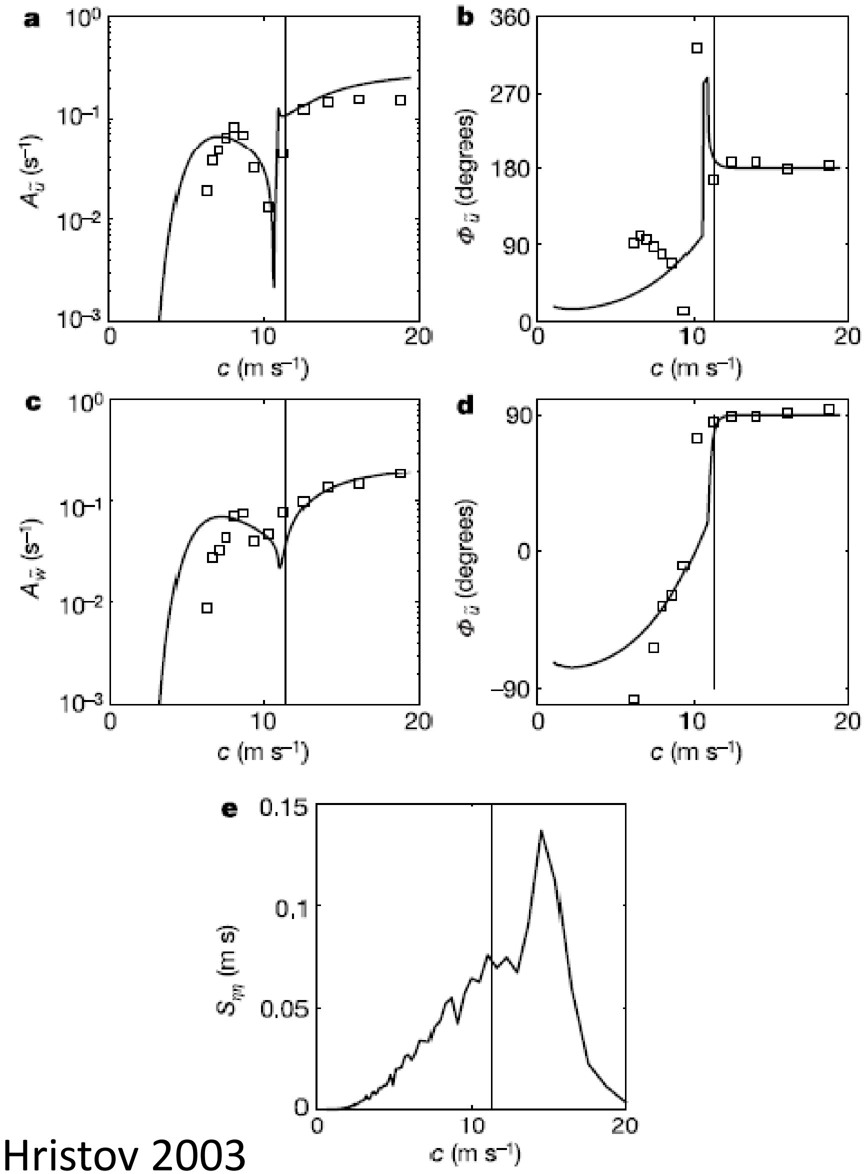
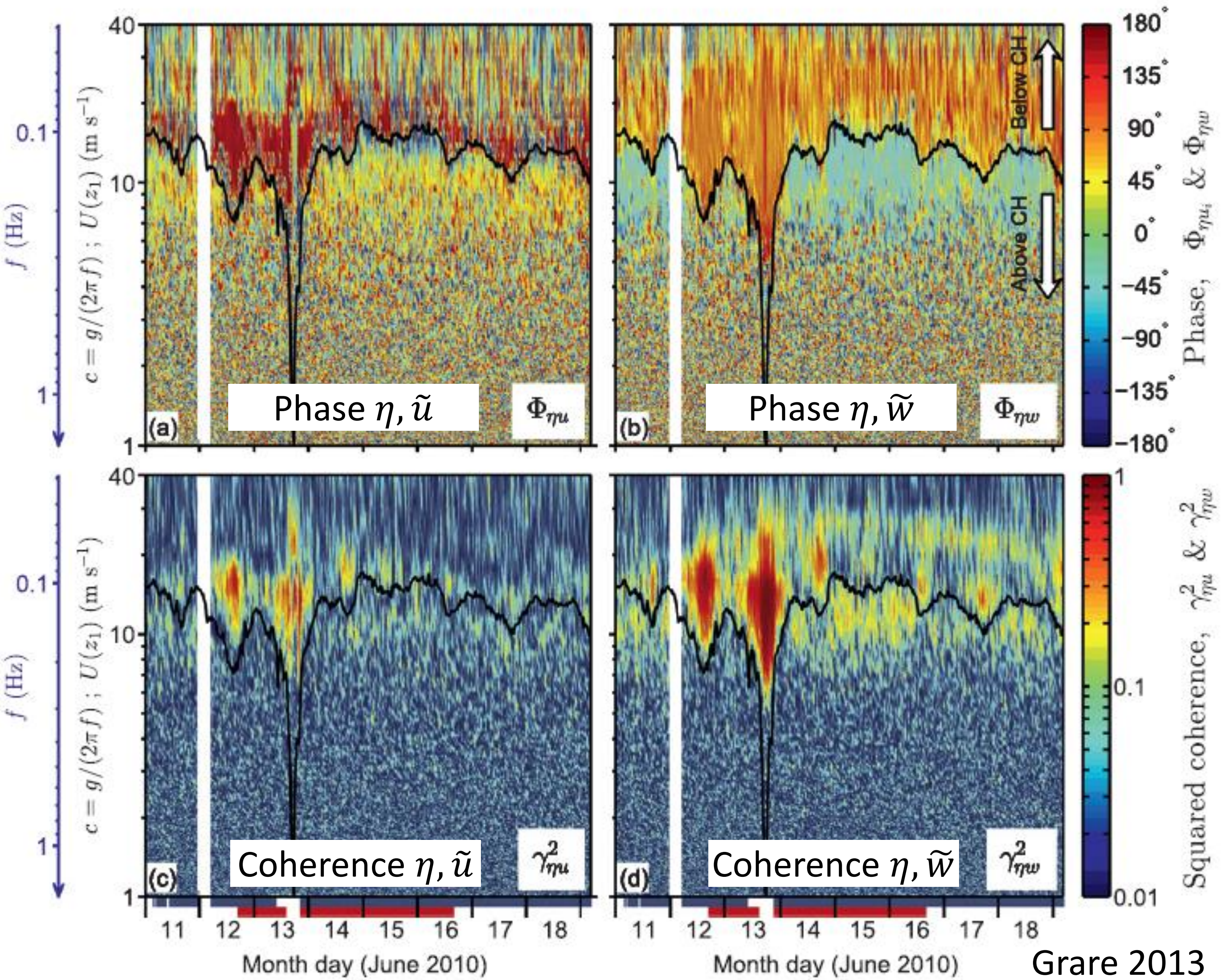


Fig. 3.3. Comparison of growth rates according to Miles' theory with observations compiled by Plant (1982). Full line: Miles'theory; open symbols: field data; full symbols and ×: laboratory data.

More recently Hristov 2003 and our group (Grare 2013 & 2018) have also shown good agreement between Miles' theory and field observations.



Hristov 2003



Grare 2013

‘New’ numerical tools (DNS, LES) have also shown the role of the critical layer.

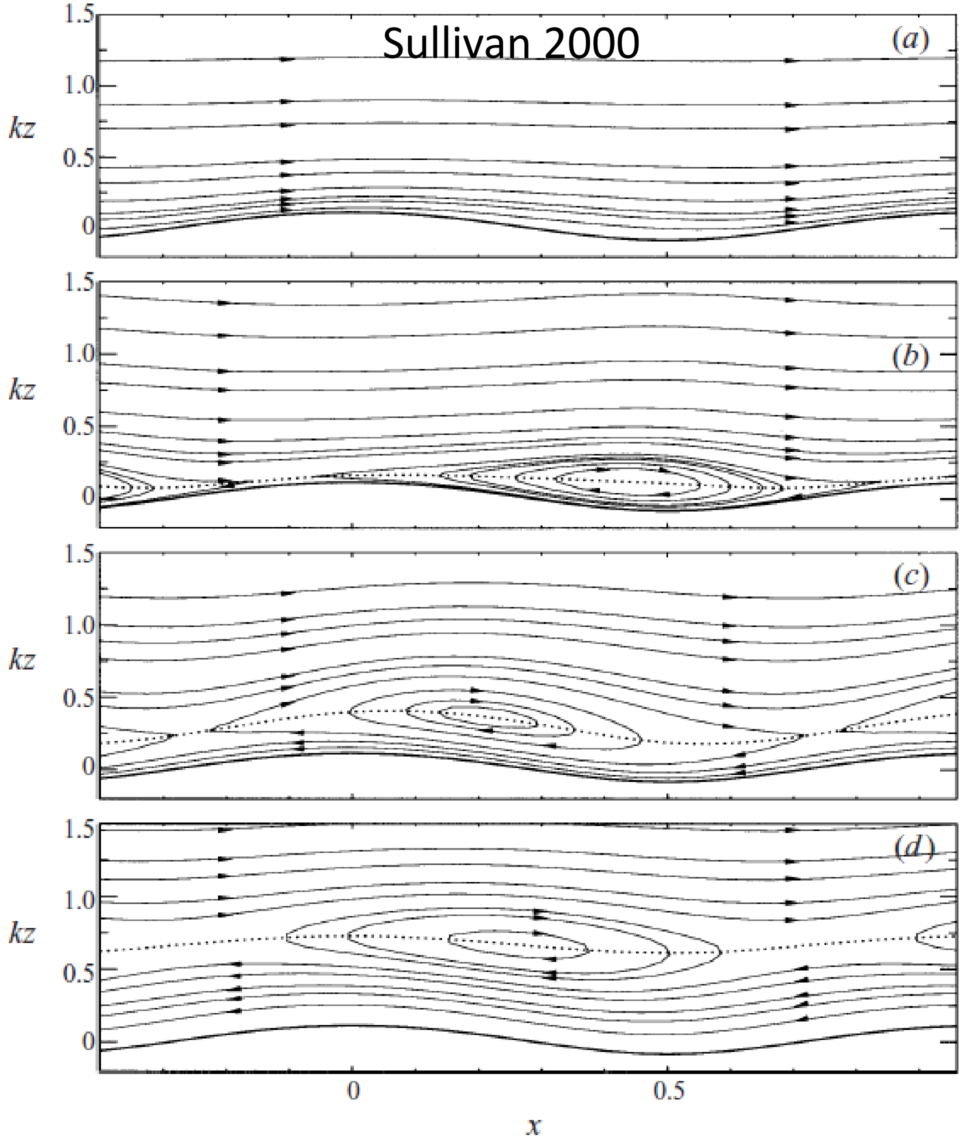


FIGURE 16. Phase-averaged streamlines over stationary and moving waves with $ak = 0.1$ in surface-fitted coordinates: (a) $c/u_* = 0$; (b) $c/u_* = 3.9$; (c) $c/u_* = 7.8$; (d) $c/u_* = 11.5$. The dotted line corresponds to the height of the critical layer where $\langle u \rangle + u_w = 0$.

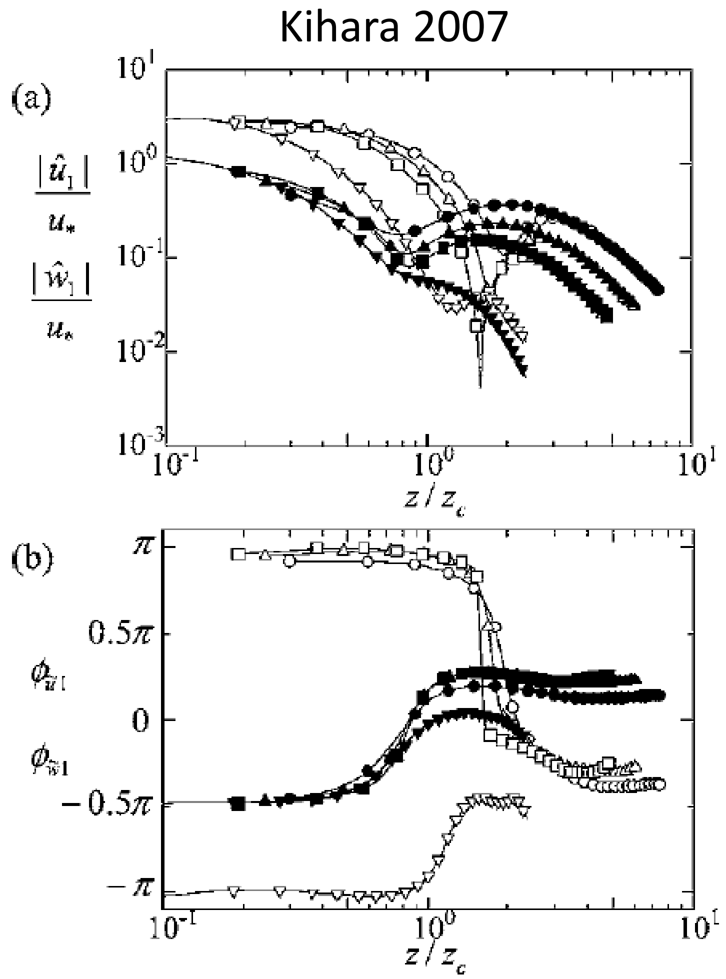
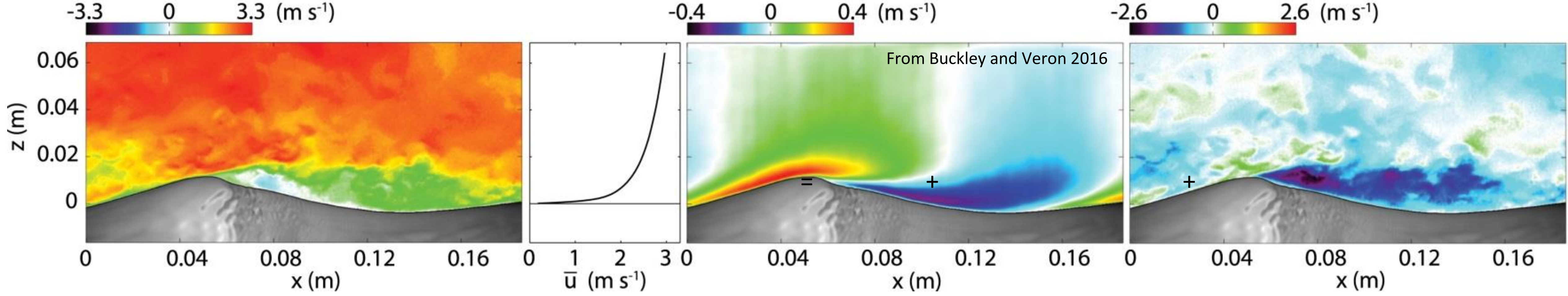


FIG. 6. Log-log plot of the amplitudes $|\hat{u}_1|$, $|\hat{w}_1|$ and the phases $\phi_{\tilde{u}1}$, $\phi_{\tilde{w}1}$ of the fundamental mode vs z/z_c for various wave ages. (○), (△), (□), and (▽) denote the $\tilde{u}$ components for c/u_*=8, 10, 12, and 16; (●), (▲), (■), and (▼) denote the $\tilde{w}$ components for c/u_*=8, 10, 12, and 16.

Definition and computation of the wave-induced fluctuations.

1. What is the wave-induced velocity ?

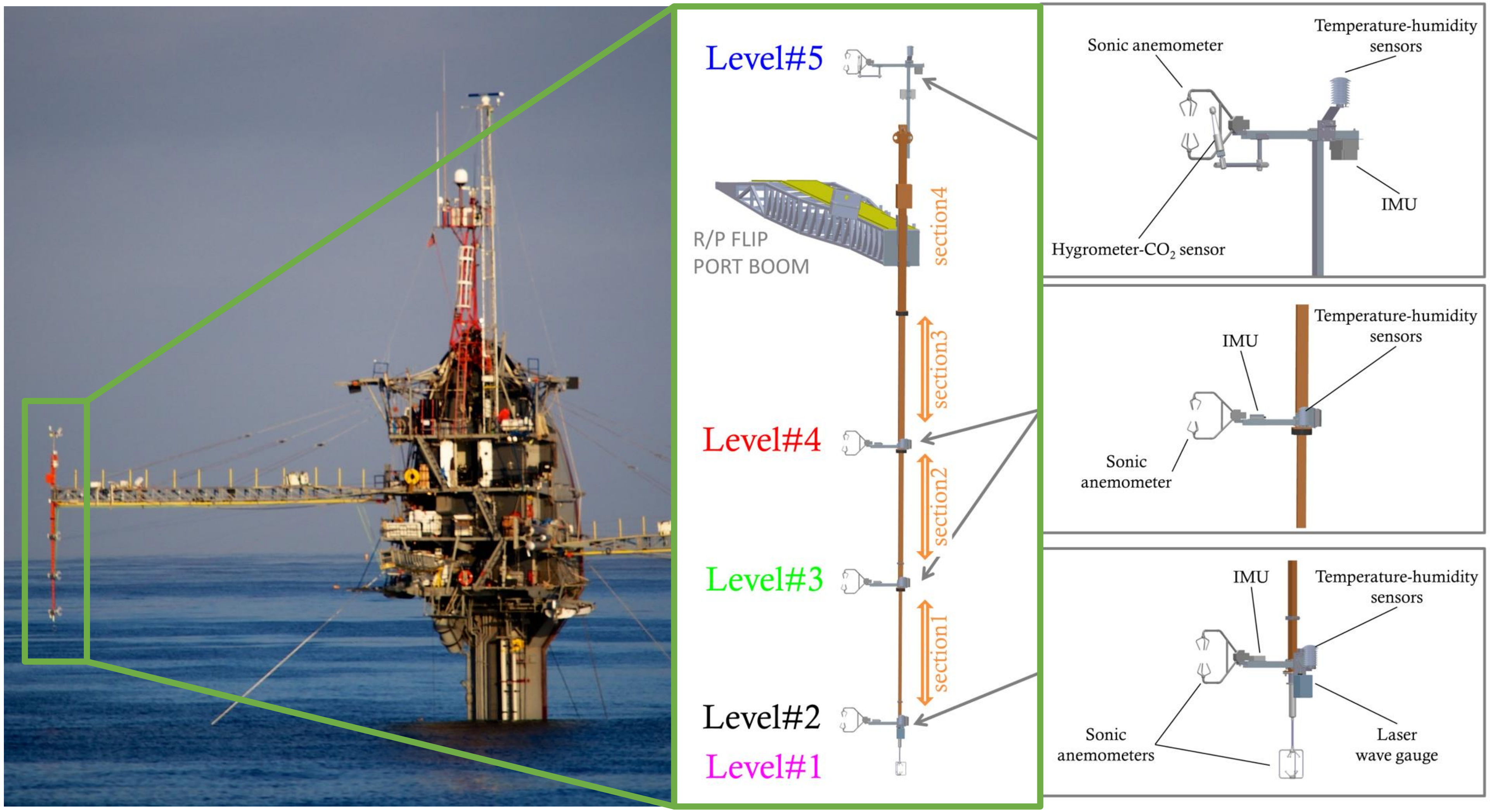
$$u(x,z,t) = \overbrace{\bar{u}(z) + \tilde{u}(x,z)}^{\langle u\rangle(x,z)\text{ is the phase averaged velocity}} + u'(x,z,t)$$



Instantaneous velocity | Mean velocity | Wave-induced velocity | Turbulent velocity

2. How to compute it for a broadband wave spectrum?
   - The complex transfer function $H_{\eta u_i}(c,z) = \tilde{u}_i(c,z)/a(c)$ gives the amplitude and phase of the wave-induced velocity $\tilde{u}_i$.
   - Nondimensionalization of the wave-induced velocities by wave orbital velocity akc.

The experimental setup on R/P FLIP during SoCal 2013



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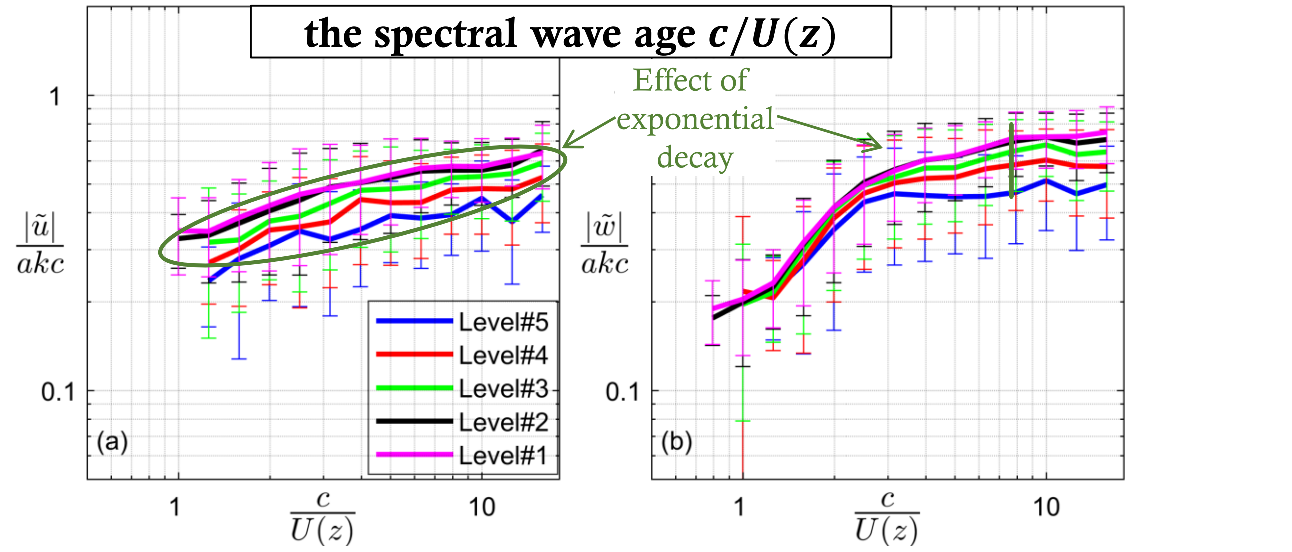
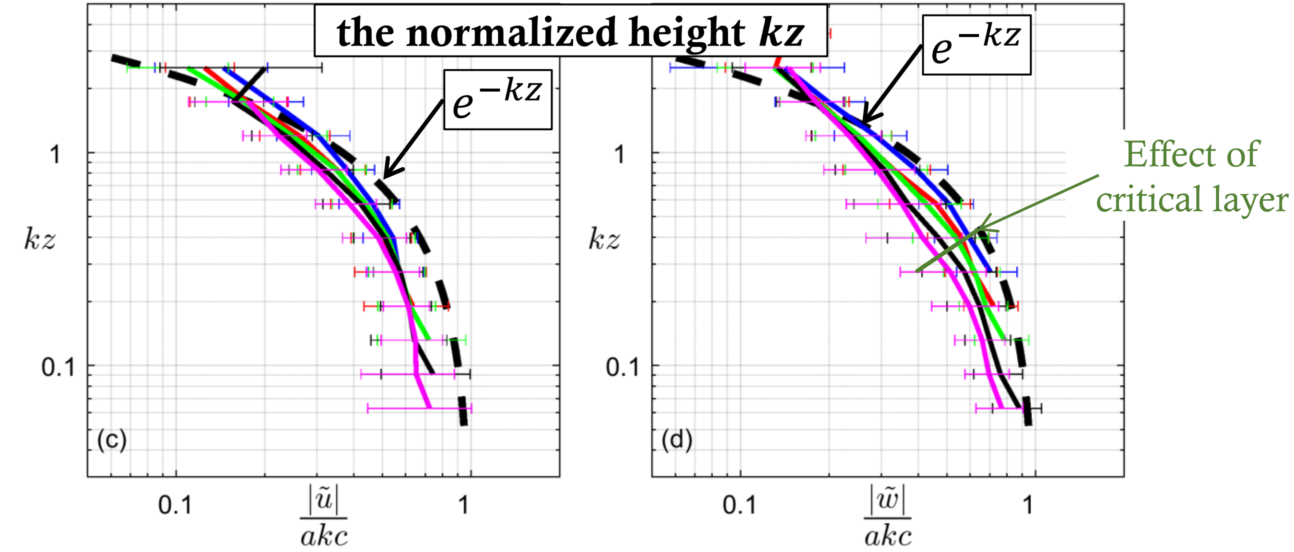
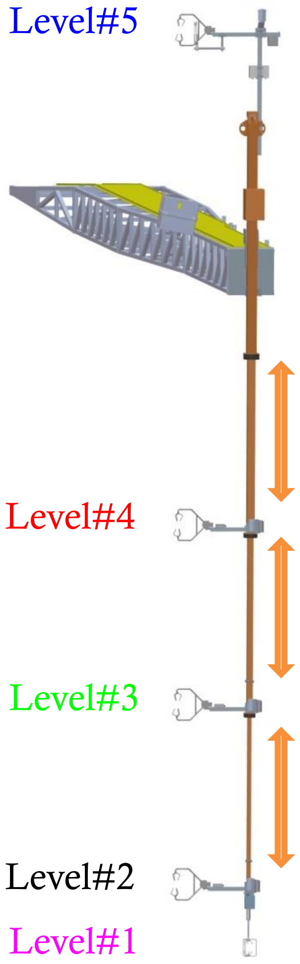
Dependence of the wave-induced velocities on:



Can we parameterize this double-dependence and collapse the data from all anemometers?





- No collapse because the data depend both on $c/U(z)$ and kz

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Parameterization of the double dependence of the wave-induced velocities on $c/U(z)$ and kz

Proposed parameterization:

$$\frac{|\tilde{u}_i|}{akc} = A_i . F_i\left(\frac{c}{U(z)}\right) . G_i(kz)$$

with $F_i\left(\frac{c}{U(z)}\right) = 1 - \beta_i \exp\left(-\gamma_i \left|1 - \frac{c}{U(z)}\right|\right)$

and $G_i(kz) = \exp(-\alpha_i kz)$

- ➢ Coefficients are determined by minimizing a squared-difference cost function between data and model.
- A_i defines the coupling between the wave-induced velocity and the wave orbital velocity.
- β_i determines the minimal value of the wave-induced velocity when $\frac{c}{U(z)} = 1$.
- γ_i adjusts how quickly the wave-induced velocity decreases when $\frac{c}{U(z)} \to 1$.
- α_i defines the deviation of the vertical profile of the wave-induced velocities from the e^{-kz} function.
- ➢ Check the validity of the parameterization by looking at the wave-induced velocity corrected for each dependence:

$$\frac{|\tilde{u}_i|}{akc} \cdot \frac{1}{G_i(kz)} = A_i . F_i\left(\frac{c}{U(z)}\right) ? \qquad \& \qquad \frac{|\tilde{u}_i|}{akc} \cdot \frac{1}{F_i\left(\frac{c}{U(z)}\right)} = A_i . G_i(kz) ?$$

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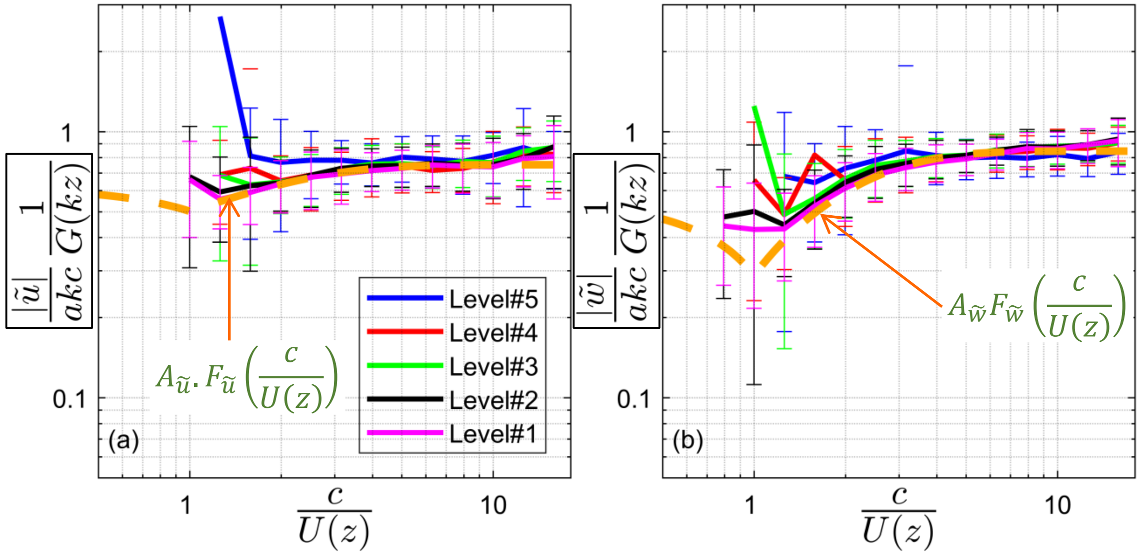
Parameterization of the double dependence of the wave-induced velocities on the spectral wave age and the normalized height



Proposed parameterization:

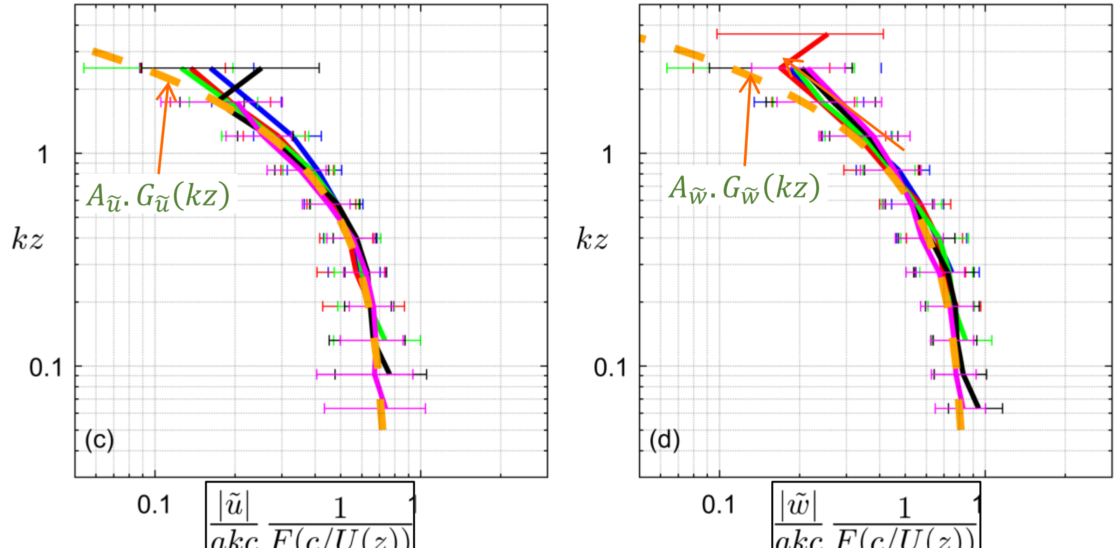
$$\frac{|\tilde{u}_i|}{akc} = A_i . F_i\left(\frac{c}{U(z)}\right) . G_i(kz)$$

with $F_i\left(\frac{c}{U(z)}\right) = 1 - \beta_i \exp\left(-\gamma_i \left|1 - \frac{c}{U(z)}\right|\right)$

and $G_i(kz) = \exp(-\alpha_i kz)$

$A_{\tilde{u}} = 0.76$; $\beta_{\tilde{u}} = 0.32$; $\gamma_{\tilde{u}} = 0.98$; $\alpha_{\tilde{u}} = 0.85$; $r^2_{\tilde{u}} = 0.63$

$A_{\tilde{w}} = 0.85$; $\beta_{\tilde{w}} = 0.66$; $\gamma_{\tilde{w}} = 1.01$; $\alpha_{\tilde{w}} = 0.83$; $r^2_{\tilde{w}} = 0.76$



- Good collapse of the data in all panels.
- Comparable coefficients for horizontal and vertical velocities.
- γ_i is very close to 1
  - → Can be constrained to 1
  - → Simpler parameterization.
- A_i and α_i smaller than 1.
- High coefficient of determination r^2.

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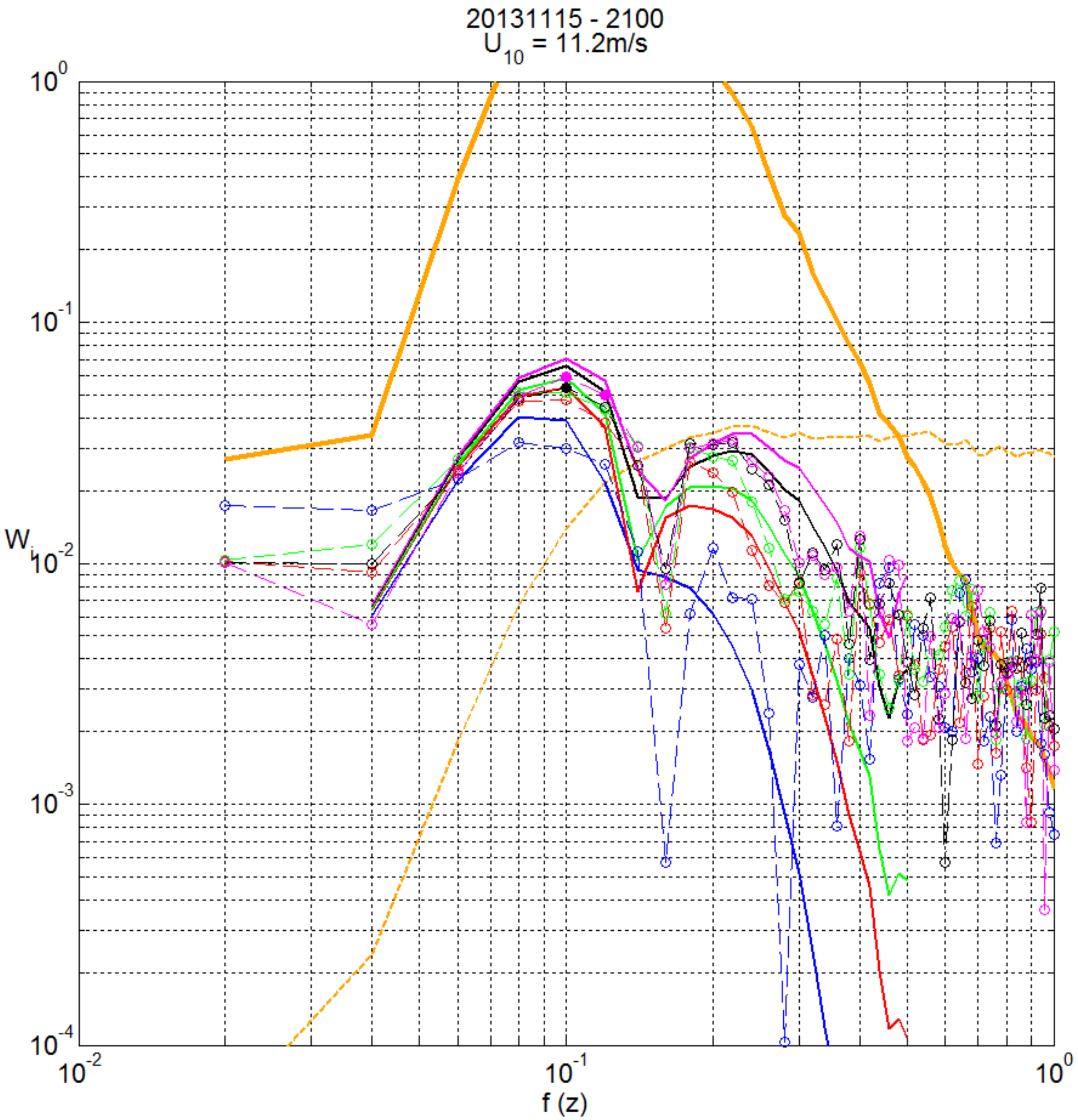
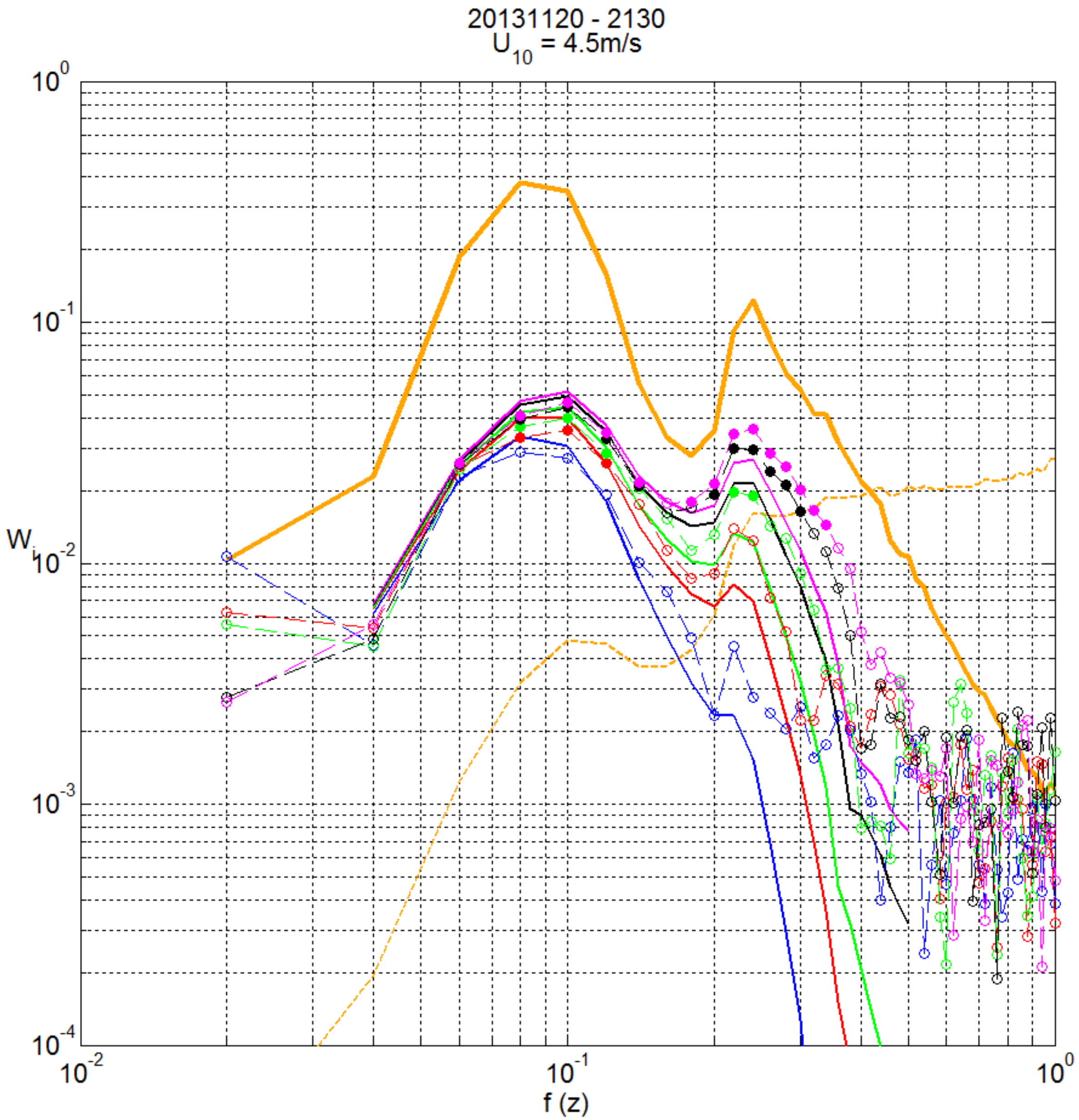
Comparison of the amplitude of the wave-induced vertical velocity

Measurements vs Miles' theory (using only $S_{\eta\eta}$ & u_*)





Limitations and controversy of the Miles' theory:

- Inviscid fluid: viscosity has been incorporated in the model by Benjamin 1959 & Miles 1959
- Effect of turbulence → many authors (Gent & Taylor 1976, Makin …, Belcher & Hunt 1993)
- Nonlinear effects such as wave-mean flow interaction → Janssen
- Dependence of the mean wind profile on the sea state → Janssen
- Waves travelling faster or against the wind (i.e. Wave-driven winds → damping of the waves?).
- There are still discrepancies on the growth rate between models and **available** measurements (Peirson & Garcia 2008). Models are roughly low by a factor 2.
  They showed that the wave slope ak is a critical missing ingredient.
- Another unstable mode between surface waves and a critical layer in the water (Young 2014)

Experimental limitations:

- Weak signal of the wave-induced velocities → hard to measure (especially in the vicinity of the critical layer)
- For the shortest waves, the critical layer is real close to the surface
- Need to make measurements closer to the surface → Buoy, WaveGlider, lab exp.