 

Surface wave mass transport

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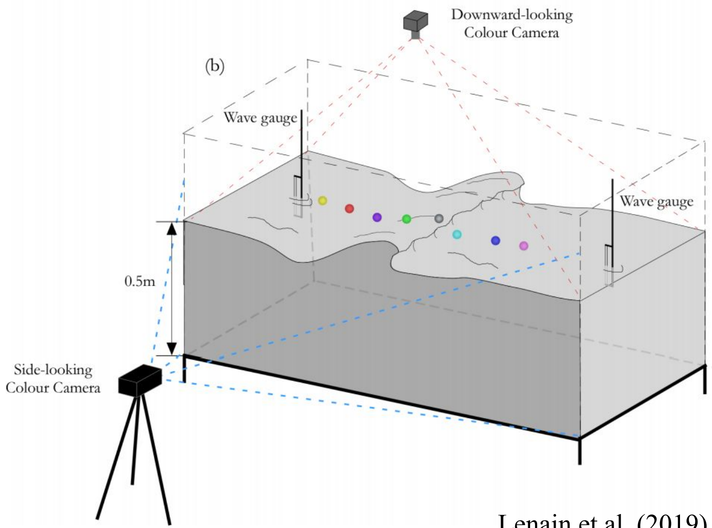
Lagrangian drift in breaking waves

- SIO Glass Channel experiments: "Lagrangian drift in breaking wave packets"
    - 30m long, 0.5m wide, 0.5m water depth
- Program paddle to create breaking wave-packets with specific parameters

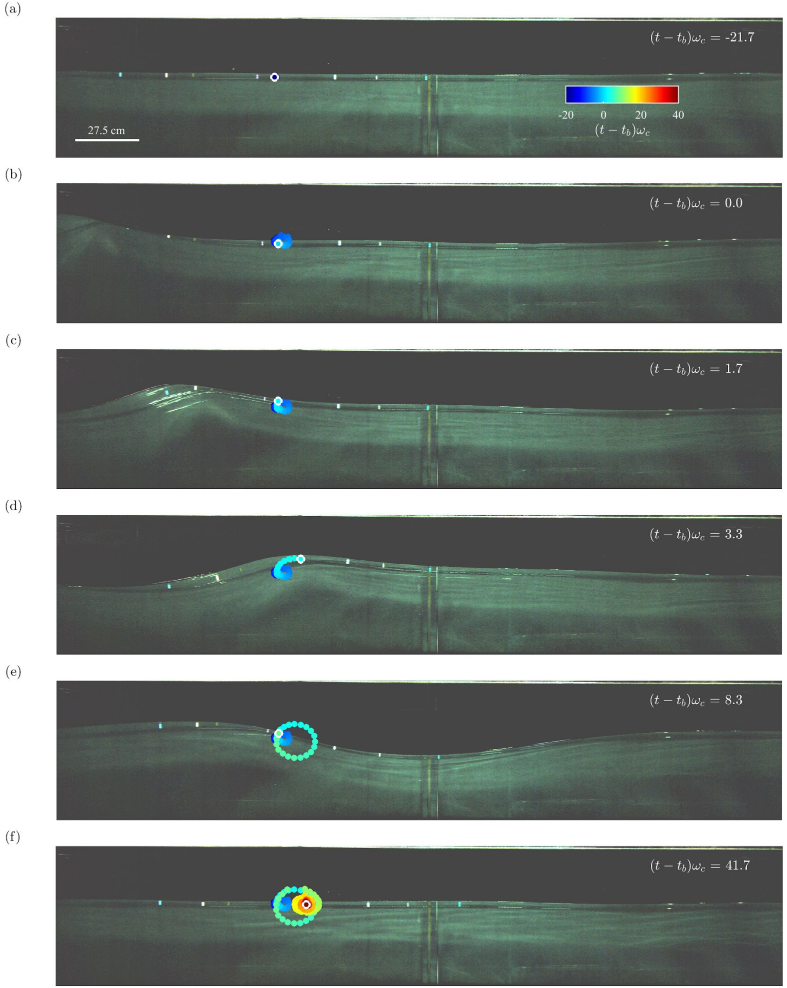
$$\eta(x,t) = \sum_{n=1}^{N} a_n \cos(k_n(x - x_b^l) - \omega_n(t - t_b^l))$$

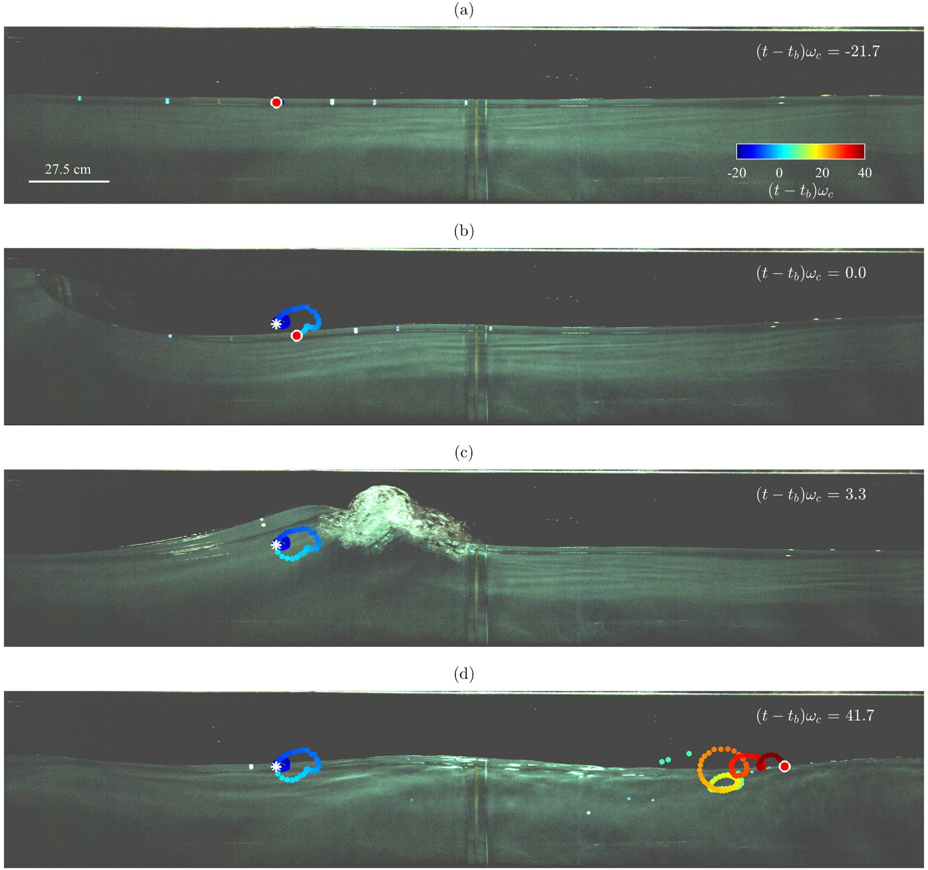
$$S = \sum_{n=1}^{N} a_n k_n$$ Maximum linear slope



Lenain et al. (2019)

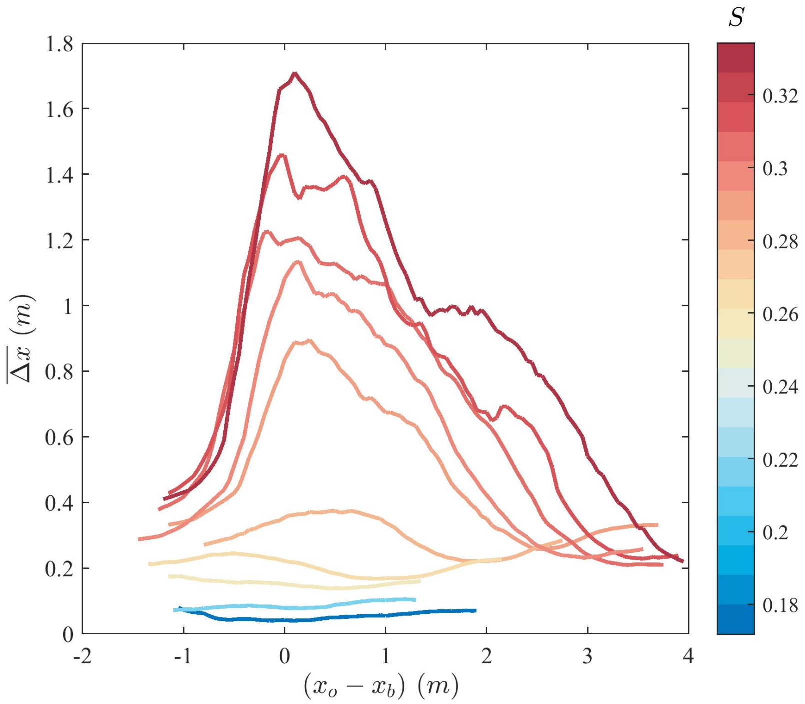
- Seed water with neutrally-buoyant particles
- Measure total drift during breaking



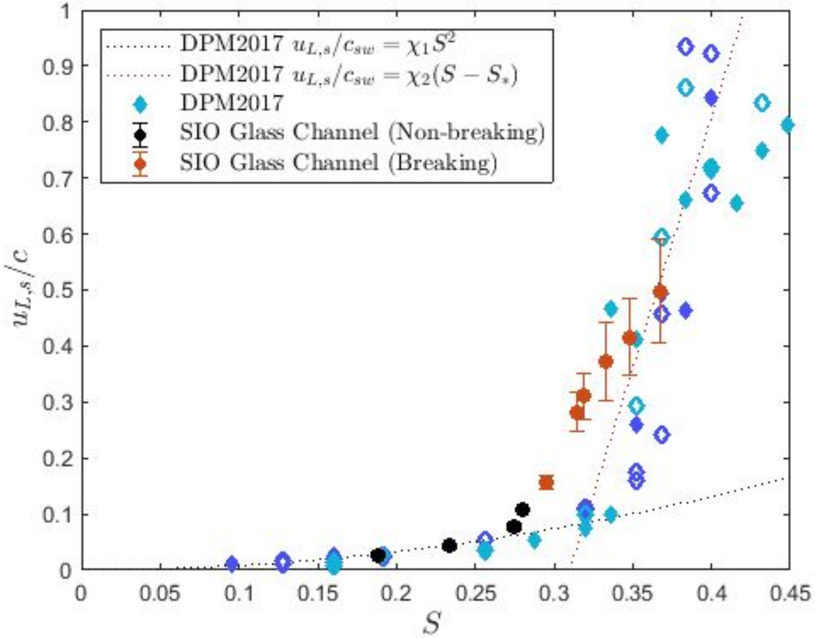


Lenain et al. (2019)

Average Lagrangian drift profiles and transport

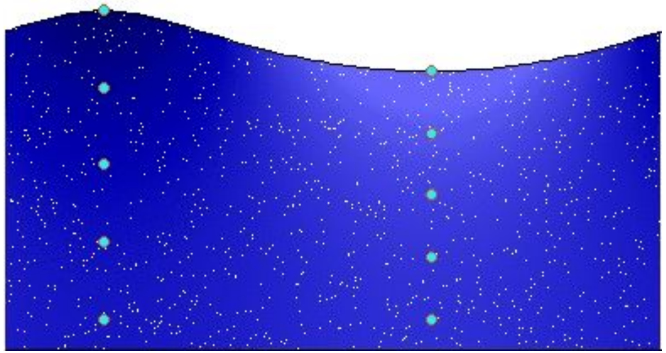


Breaking packets show asymmetric Lagrangian drift profile



Different scaling with S depending on breaking, qualitative agreement with DNS of Deike et al

Stokes Drift for a single wave



- George Gabriel Stokes 1847
- Follow derivation of O.M. Phillips 1966:
  - Taylor expansion around particle's initial position
  - Ignore terms beyond first order of lagrangian displacement (2nd order in wave slope)
- Resulting mean displacement velocity:

$$\frac{\overline{u_L}}{c_p} = (ak)^2 \frac{cosh(2k(z_o + d))}{2sinh^2(kd)}$$

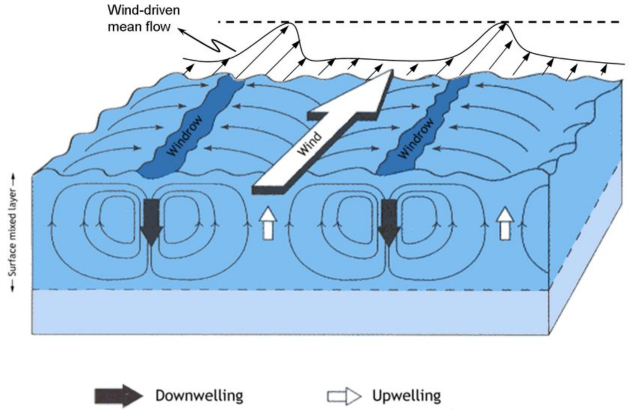
Arbitrary depth *d*

$$\frac{\overline{u_L}}{c_p} = (ak)^2 e^{2kz_o}$$

Deep water

Animation claimed by Wikipedia user Kraaiennest

Significance of Stokes drift at larger scales: van den Bremer and Breivik, 2017



1) Mass transport on coasts
   a) movement of sediment
   b) opposing rip-currents and undertow
2) Forcing in ocean models — Navier-Stokes equation
   a) Langmuir turbulence
      i) CL vortex force
      ii) turbulent mixing in upper ocean
   b) Coriolis-stokes forcing
   c) McWilliams and Restrepo (1999): 40% correction to Ekman transport at higher latitudes
3) Transports matter, salinity and heat in water
   a) pollution, oil, trash
      i) any attempts to account for contaminated surface in oil slicks?

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + 2\boldsymbol{\Omega} \times \boldsymbol{u} + \underbrace{2\boldsymbol{\Omega} \times \boldsymbol{u}_{\mathrm{SD}}}_{\text{Coriolis–Stokes force}} + \boldsymbol{\nabla}\underbrace{(\pi + \boldsymbol{u} \cdot \boldsymbol{u}_{\mathrm{SD}})}_{\text{Stokes-corrected pressure}}$$

$$= \underbrace{\boldsymbol{u}_{\mathrm{SD}} \times (\boldsymbol{\nabla} \times \boldsymbol{u})}_{\text{Craik–Leibovich vortex force}} + \nu \nabla^2 \boldsymbol{u},$$

Equation from van den Bremer and Breivik, (2017)
Figure from Akan (2012)

Kenyon 1969: Stokes Drift for Random Gravity Waves

- Method: adds drifts of each component in surface wave spectrum.
  - Not dependent on deep water approximation
- Given an energy spectrum (directional or unidirectional), can estimate the stokes drift.
- Applies method to empirical, one-dimensional spectra
- Not compared to measurements of Stokes drift
  - Difficulties directly measuring Stokes drift in the ocean and in wave tanks.
- Finds Stokes drift to be 1-5% of wind speed measured at 19.5m above ocean surface.

$$F(\mathbf{k}) = 2\rho g \langle \eta_k \eta_k^* \rangle$$

$$U(z) = 1/\rho \iint_{-\infty}^{\infty} F(\mathbf{k})\mathbf{k} \frac{2k}{\omega(k)} \frac{cosh2k(z+d)}{sinh(2kd)} d\mathbf{k}$$

Kenyon 1969: conditions for the method

- Conditions:
    1. Fully developed seas without breaking (breaking may increase Stokes drift by 30% - Pizzo et al 2019)
    2. Small wave slopes
    3. Horizontally homogeneous and statistically stationary: spectrum doesn't depend on x or t, allows summing over individual components.
    4. Inviscid fluid: Likely small effect on surface drift in deep water, however, it affects the drift's gradient right below the surface (Phillips 1966: Section 3.4 & Longuet-Higgins 1960)
    5. Clean water

Application of the method

- Uses spectra of the form given by Pierson and Moskowitz:

$$f_n(\omega) = (\alpha_n \rho g^3 / \omega^5) e^{-\beta_n (g/W\omega)^n}$$

- Returns drift at surface as percentage of the given wind speed
  - Sensitivity of this measurement to the fitting parameter n?

$$\frac{U_n(0)}{W} = \begin{cases} 3.58\% & n = 2 \\ 2.09\% & n = 3 \\ 1.57\% & n = 4 \end{cases}$$

- Finds Stokes drift as a function of depth for n=2
- “Tomczak indicates … aside from regions of strong permanent currents, … values for the ratio of total surface drift to wind speed vary widely from about 1.4 to 4.3%”

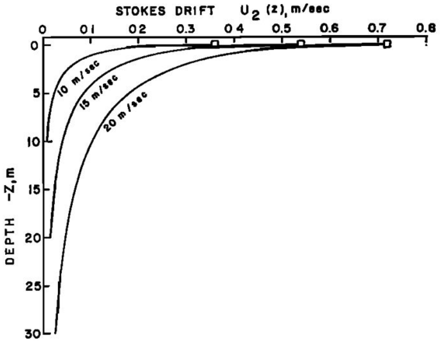


Fig. 2. The Stokes drift velocity $U_2(z)$, based on the spectrum in Figure 1, as a function of the depth z for the three wind speeds 10, 15, and 20 m/sec.

Possibly interesting: viscosity and Stokes drift

- "The existence of a non-vanishing viscosity of the water results in the development of streaming motions—second-order mean velocity fields—which are germane in questions of the mass transport in waves." — Phillips 1966, Section 3.4
- Induced streaming has singular dependence on viscosity.
  - Viscosity implies energy loss, this implies momentum loss, this implies a strain.

$$\nu \neq 0 \implies \dot{E} \neq 0 \implies \dot{M} \neq 0 \implies S \neq 0$$

  - This strain induces an eulerian mean-flow.
- Doubles the gradient of drift velocity right below the free-surface.
- Better understood at bottom boundary layer.