WIND WAVES IN THE COUPLED CLIMATE SYSTEM

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Outline

- Intro
- A brief survey of deep-water surface wave theory
  - Linear waves
  - Wave instabilities
  - Surface tension
  - Wave breaking
- Discussion questions

Air-sea boundary layer

The historical view:

"Meteorologists consider the ocean as a wet surface. Oceanographers consider the atmosphere as a place where wind blows." –Erik Mollo-Christensen

Air-sea boundary layer

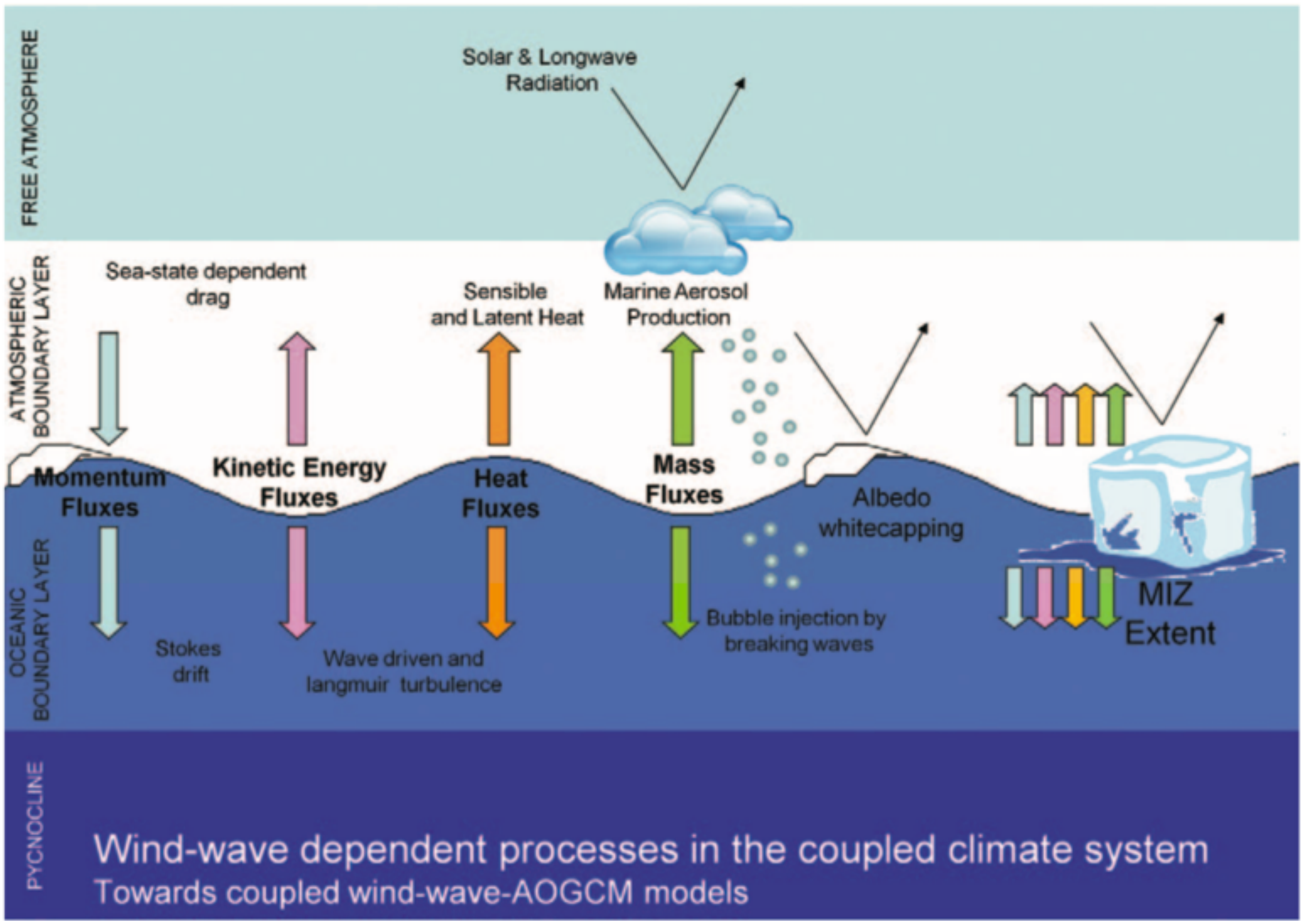
The modern view:

Ocean and atmosphere **strongly coupled**: understanding their **interactions** crucial for better description of weather and climate

Air-sea boundary layer

The modern view:

These **interactions** are controlled by the dynamics of the separation surface: **waves**.



Cavaleri et al. (2012)

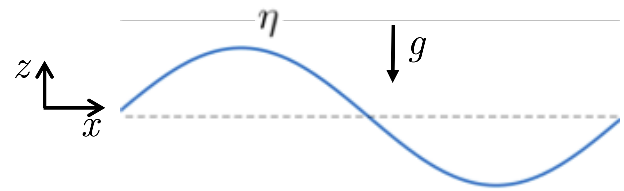
- Of the solar radiation incident on the atmosphere, 40% is absorbed in the first 10 m of the ocean; more in coastal waters.
- The first 10 m of the water column has the same weight as all the atmosphere above.
- The first 2.5 m of the water column has the same heat capacity as all the (dry) atmosphere above.

Gill (1982)

After the overall radiation balance, these exchange processes are the next priority in providing predictions of the climate system from seasons to centuries.

Why are these impacts not included in more models of upper ocean dynamics?

Theory



Start with quiescent air/water system. Act on it with conservative forces

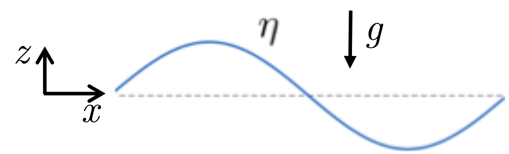
$$\phi_{xx} + \phi_{zz} = 0, \qquad \mathbf{u} = (\phi_x, \phi_z).$$

$$\phi_t + \frac{1}{2}\left(\phi_x^2 + \phi_z^2\right) + gz = 0; \qquad \eta_t + \phi_x \eta_x = \phi_z; \qquad @\ z = \eta.$$

$$\phi_z \to 0 \quad \text{as} \quad z \to -\infty.$$

Theory

- Equations governing two-dimensional irrotational inviscid deep-water surface gravity waves:



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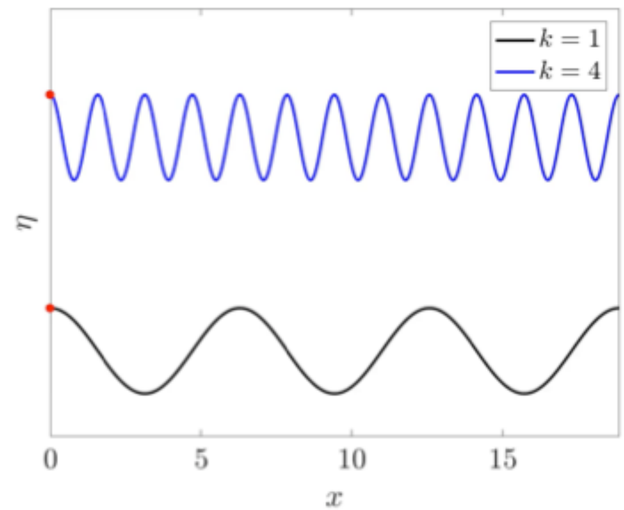
- Full water wave equations are analytically intractable.
  - Nonlinear
  - More severely, surface boundary conditions evaluated at unknown dependent variable $\eta(x,t)$.
- Therefore we must simplify the governing equations to make analytical progress.

Dispersion

- Irrotational inviscid deep-water **linear** surface gravity waves ($|k\eta| \ll 1$)
  - $\eta = a\cos(kx - \omega t)$, $\phi = \frac{a\omega}{\mathrm{k}} \sin(kx - \omega t) e^{kz}$,

- Dispersion relationship: $\boldsymbol{\omega^2 = gk}$
  - Connects space/time

- Phase speed: $c = \frac{\omega}{k} = \frac{g}{\omega} = \sqrt{g/k}$

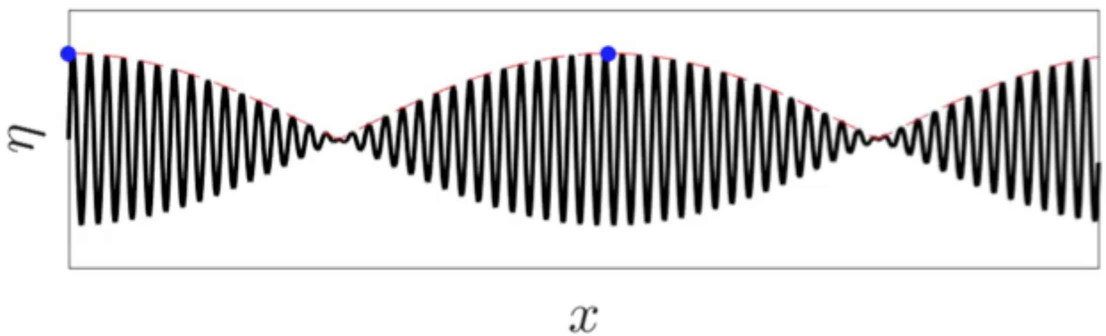


- **Deep-water surface gravity waves are dispersive**

Group velocity

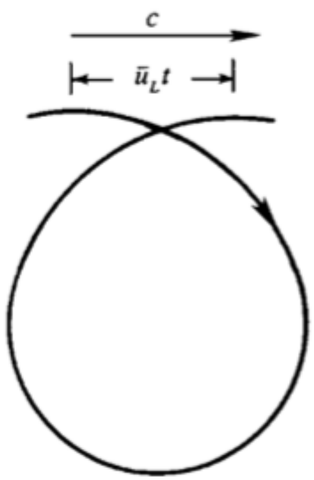
- Group velocity: $c_g = \dfrac{\partial \omega}{\partial k} = \dfrac{\omega}{2k} = \dfrac{c}{2}$
    - **Speed at which energy travels**

Blue: Phase velocity
Red: Group velocity

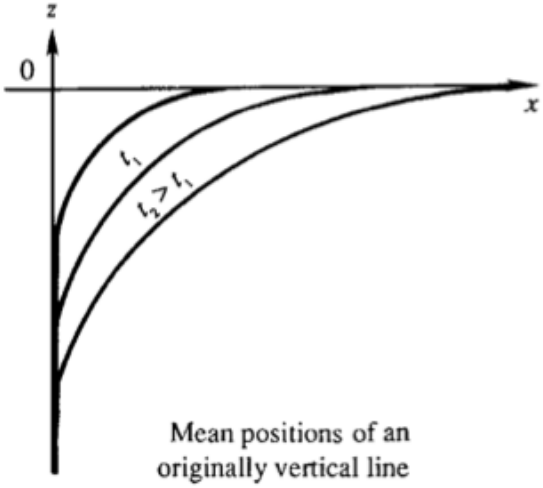


Stokes drift

- Particle orbits are not closed circles, but are slightly (to second order) elliptic
- Leads to a net drift in the direction of wave propagation: Stokes drift
  - Dynamical implications (vortex force; Langmuir circulations)



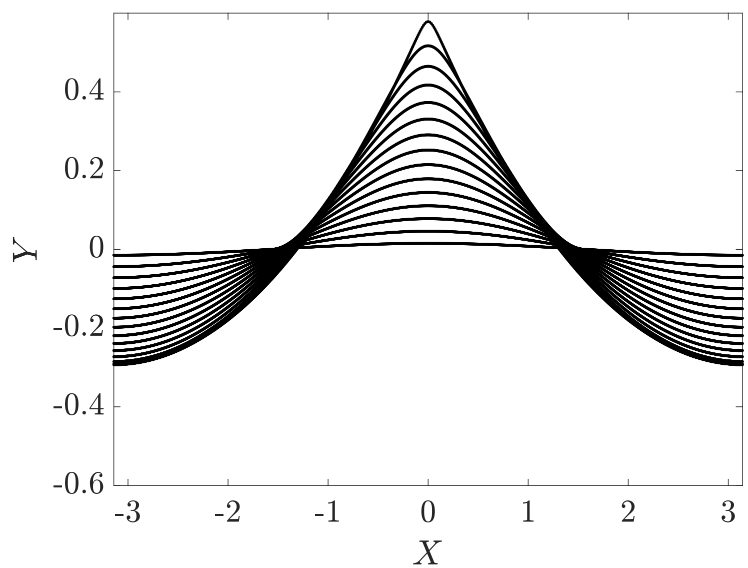
Particle orbit



Mean positions of an originally vertical line

Beyond linear theory: Stokes waves

- Permanent progressive waves (i.e. Stoke waves; dynamic equilibrium).
  - Assume $\eta = \eta(x - ct)$



Increasing slope

To second order, the phase speed c= $\sqrt{\frac{g}{k}}\left(1 + \frac{1}{2}(ak)^2\right)$

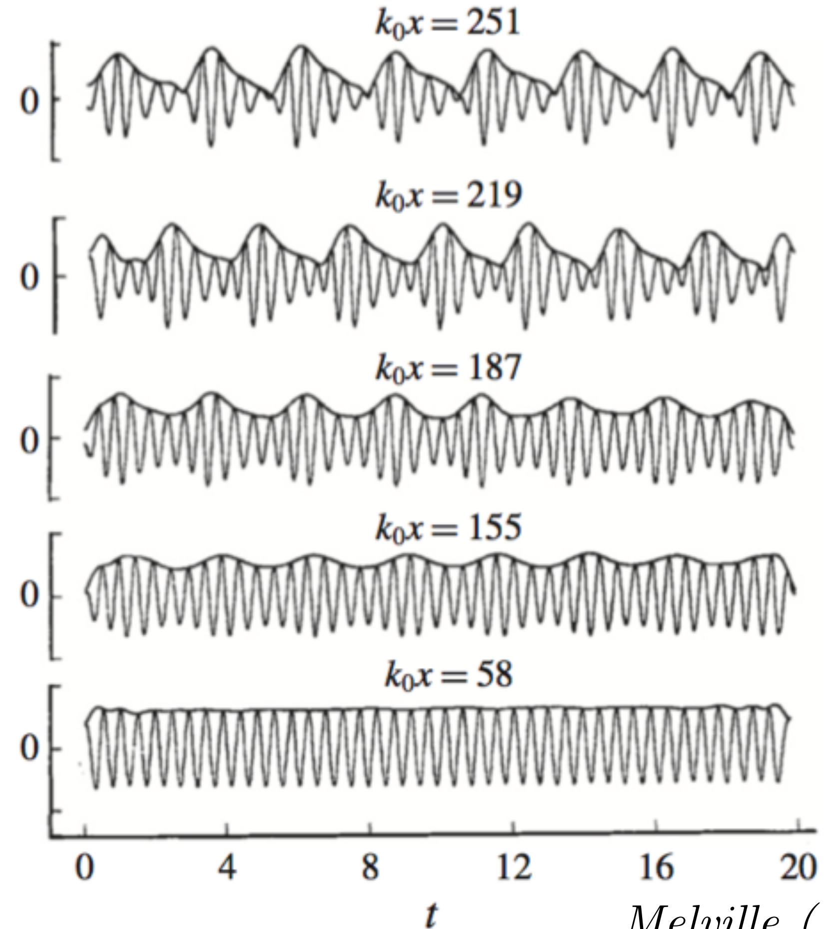
Limiting Stokes wave

In watching many years ago a grand surf which came rolling in on a sandy beach near the Giant's Causeway, without any storm at the place itself, I recollect being struck with the blunt wedge-like form of the waves where they first lost their flowing outline, and began to show a little broken water at the very summit. It is only I imagine on an oceanic coast, and even there on somewhat rare occasions, that the form of waves of this kind, of nearly the maximum height, can be studied to full advantage. The observer must be stationed nearly in a line with the ridges of the waves where they begin to break.

Stokes (1880)

Benjamin-Feir Instability

- Small slope Stokes waves are unstable to (long) perturbations (Benjamin-Feir Instability/subharmonic instability)
  - **Benjamin & Feir (1967);** Lighthill (1965); Whitham (1967); Zakharov (1968)
- Maximum growth rate: $\omega(ak)^2/2$



Melville (1983)

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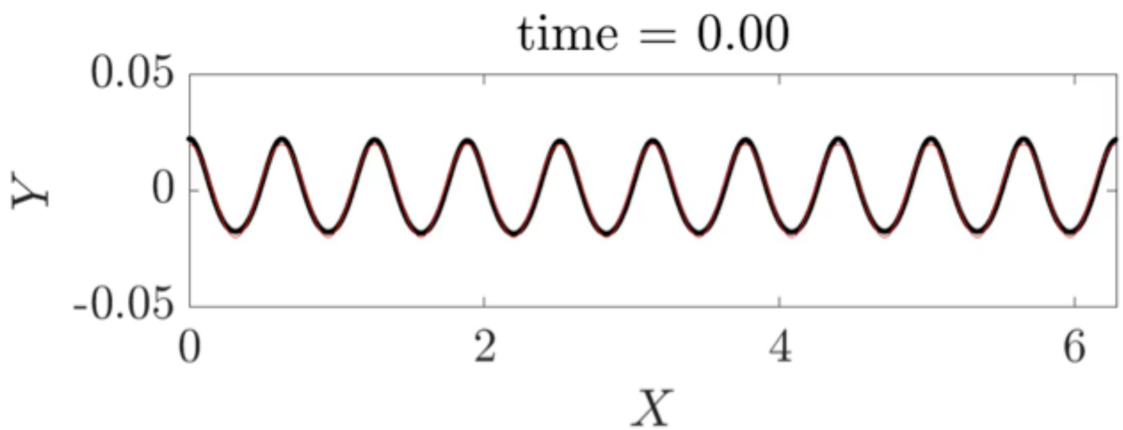
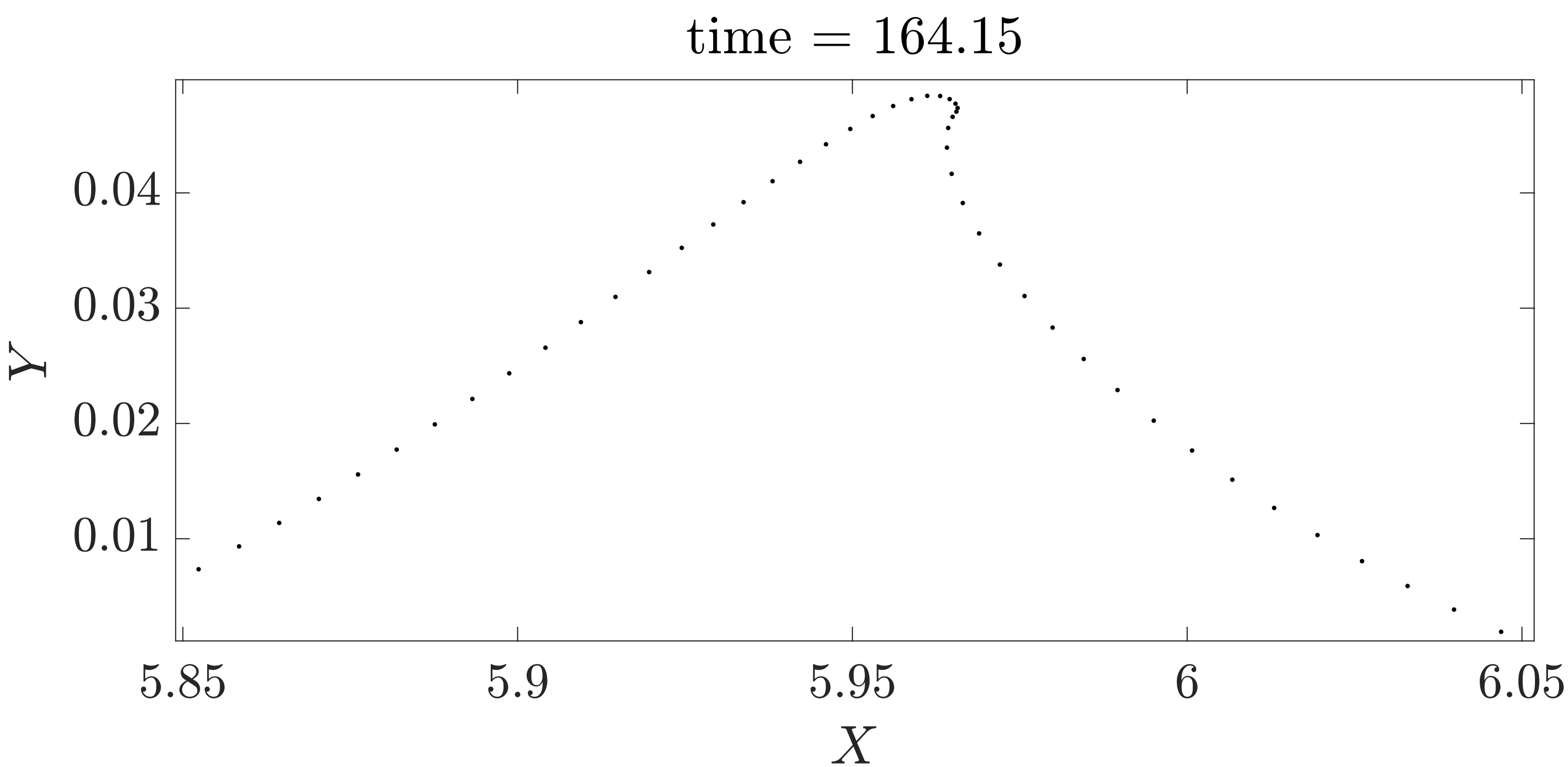
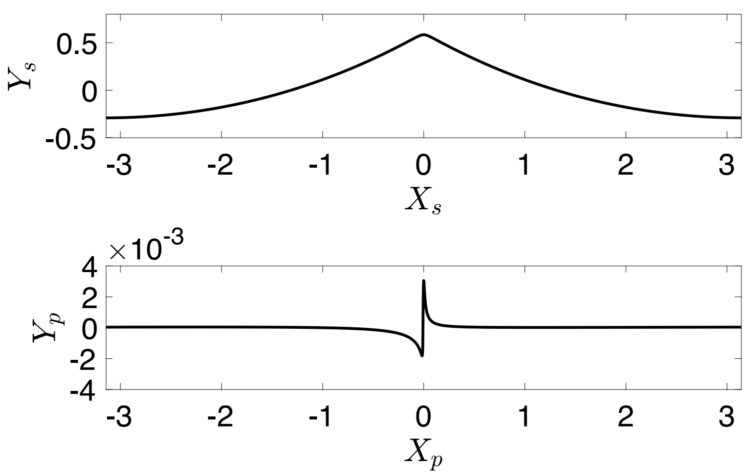


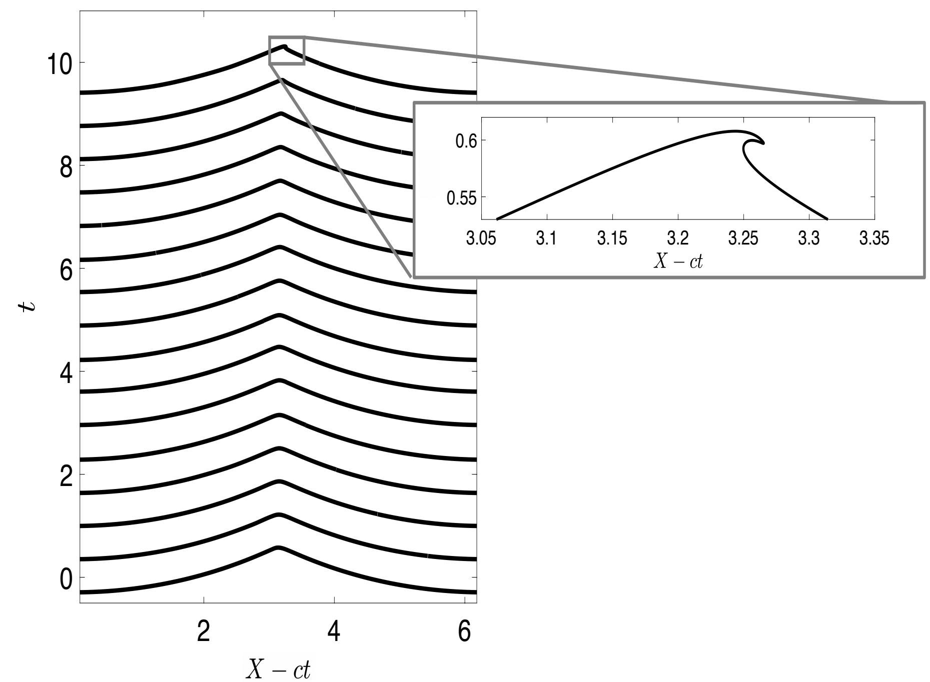
Evolution to breaking



Superharmonic instability

- Steep Stokes waves are unstable to perturbations of same length scale as underlying wave (LH 1978: **crest instability**).
- Explosive instability (can be orders of magnitude larger than BFI).





Discussion

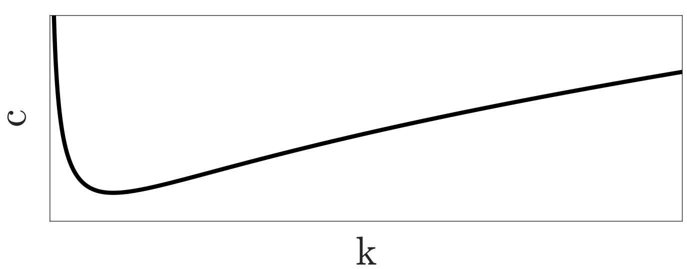
- Do nonlinearities *matter* at sea?
  - Assumption: wave field varies sufficiently rapidly in space and time to be considered random

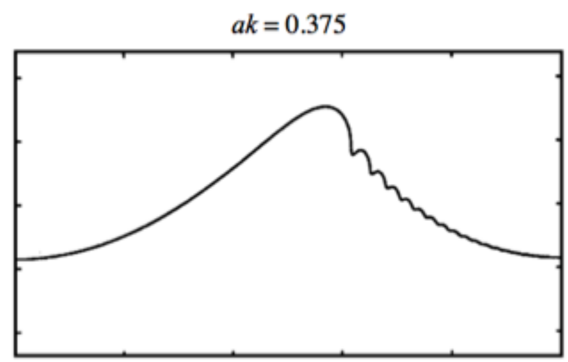
Video: Laurent Grare

Parasitic capillary waves

- Surface tension becomes important when free surface curvature becomes large (e.g. at the crest of a larger Stokes wave)
- Capillary waves ride on front face of underlying surface gravity wave
    - Highly dissipative

$$c = \sqrt{\frac{g}{k} + Tk}$$





Cox & Munk (1954)
Longuet-Higgins (1963)
Fedorov & Melville (1998)

Wave breaking

- Irrotational inviscid equations "break down"

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$$\phi_z \to 0 \quad \text{as} \quad z \to -\infty.$$

- **Vorticity generation: free surface curvature, topological changes to the fluid, baroclinic mechanism**

Luc Lenain

How to proceed?

Complexities of studying wave breaking

- Breaking is a two-phase turbulent unsteady flow, making problem out of reach using known **analytical** techniques.

- Scales of breaking range from O(1)mm (bubbles and capillary waves) to O(10)km (groups of swell), i.e. 7 orders of magnitude.
    - **Numerical simulation** far beyond current capabilities.

- Difficulty of making **measurements at sea** due to intermittency and nonlinearity of breaking, and large forces on instruments.

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Photo: Brendan G

20 second swell period, 10 waves in a group corresponds to a wave group that is over 6km long.

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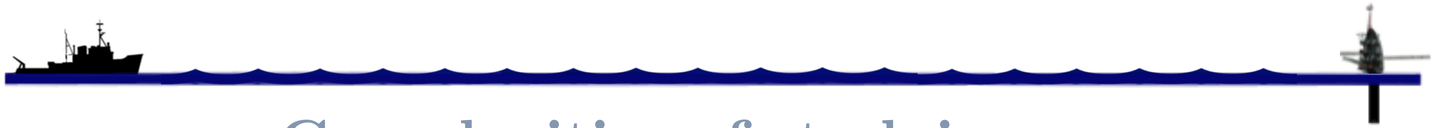

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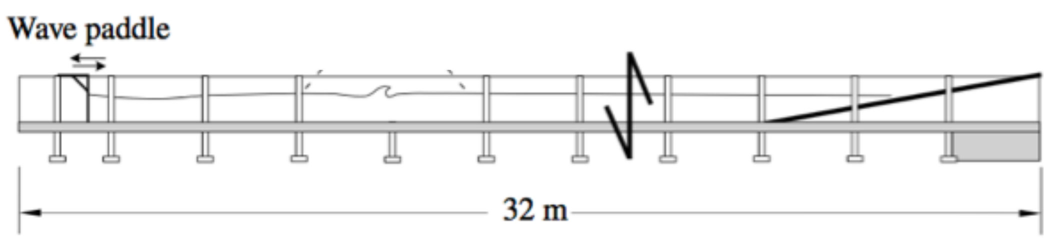
    - **Direct numerical simulation** over this *entire* range beyond current capabilities.
- Difficulty of making **measurements at sea** due to intermittency and nonlinearity of breaking, and large forces on instruments. Plus, there is the **cost.**

Wave breaking: Laboratory experiments

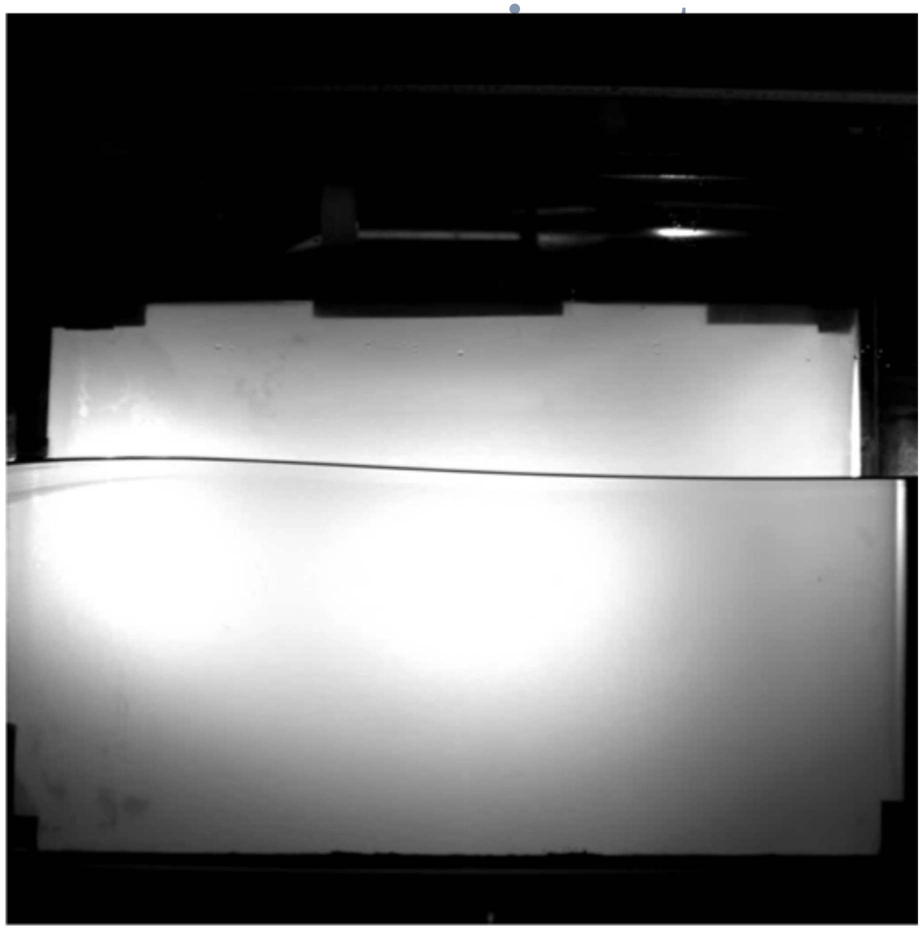
To isolate features of breaking in controlled environment, create breaking waves in the **laboratory**.

Wave channel at SIO



- 32m long
- $\frac{1}{2}$ m wide
- Filled with 60cm of water

Wave breaking: Laboratory



4 seconds in
real time

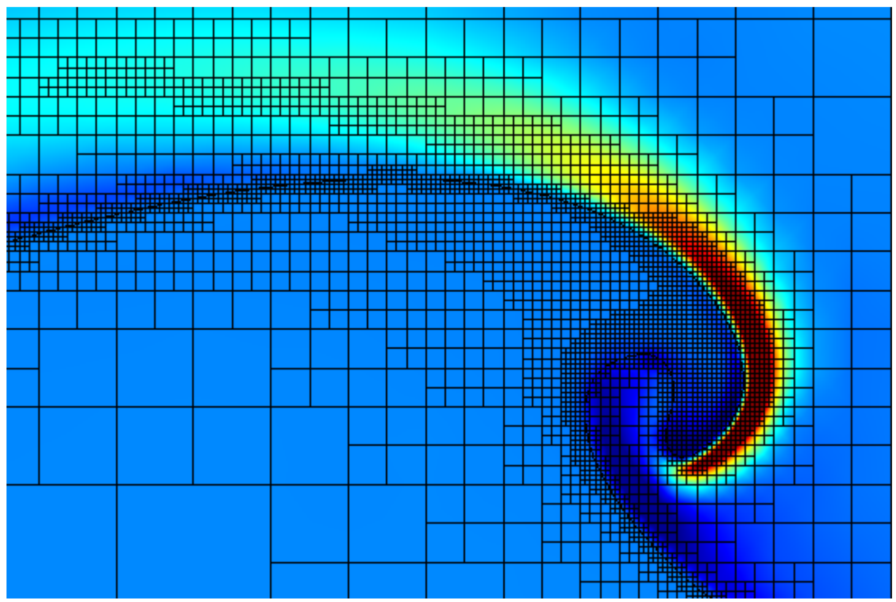
- **Bubble/spray generation**
- **Vorticity generation**

Video: Luc Lenain

Numerical experiments

(L. Deike)

- Direct Numerical Simulations (DNS) of 2d Navier-Stokes using open-source solver Gerris
- Two phase flow with surface tension *(Popinet 2009)*
    - Adaptive quad/octree discretization



Reynolds number: 40,000
Resolution 10^{-1} mm

Numerical experiments

Discussion questions

Big picture:

•Why are we talking about this now?

•Relevant campaigns?

•What are the major open questions? (How is the field moving to address these?)

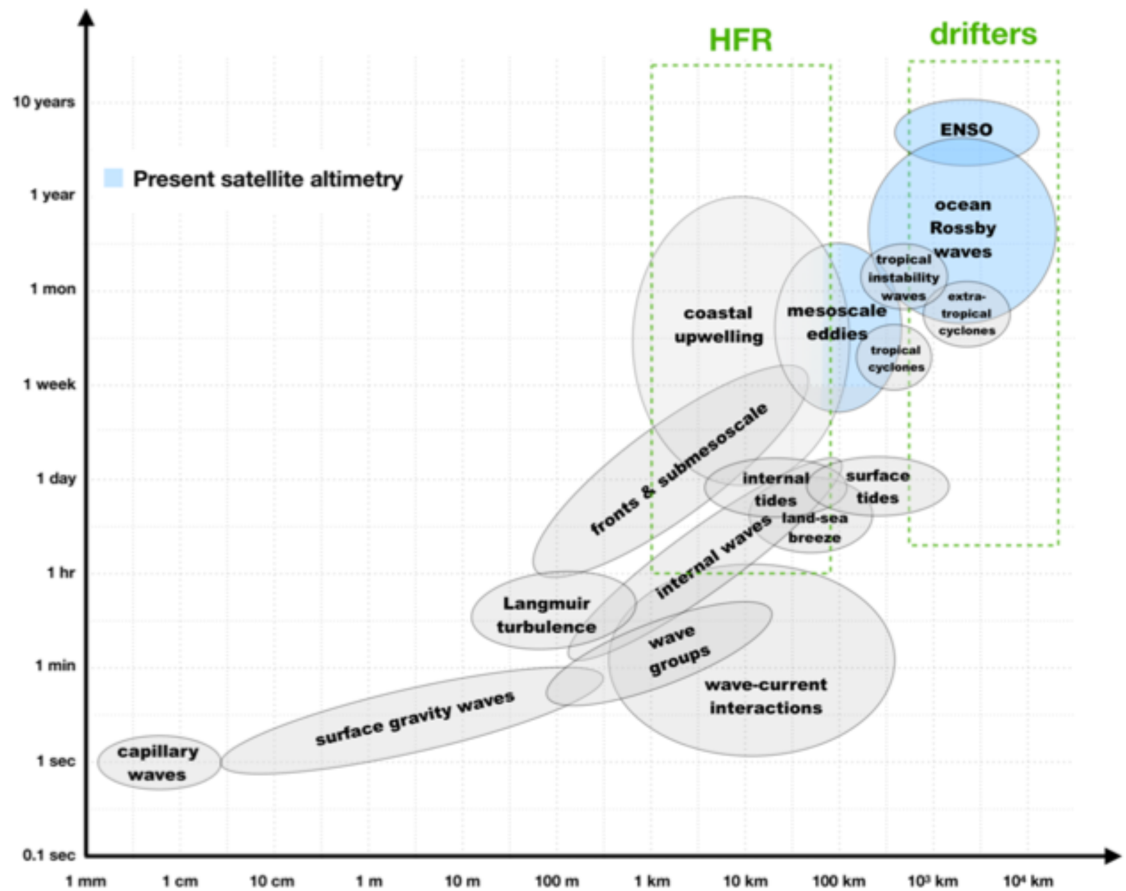
•Coupling across scales? (Show plot)

More specifically:

•Which (attainable) wave information is needed to better parametrize air-sea processes? What do sub-orbitals/satellites get right? When are higher moments needed?

•Wave breaking generates the wind-driven currents. Does our parametrization of this process (i.e. as a surface stress), matter for larger scale dynamics? How so? Do these larger scale models converge when the sub-grid scale behavior is parameterized in different ways?

Discussion questions



Villas Boas et al. (2019)

A brief history

- Newton (1687; Principia) proposed frequency of deep-water waves scales with root of inverse wavelength.

- Euler (1757): Inviscid equations of motion in hydrodynamics

- Lagrange (1781, 1786): Linear equations of motion for surface waves

- Cauchy/Poisson (1825): Initial value problem for fluid with free surface
  - (Gerstner 1802: Exact (rotational) wave solution in Lagrangian frame)

- Stokes (1847, 1880): Finite amplitude irrotational waves

Wikipedia

A brief history

- World War II
  - Sverdrup and Munk (1947)
  - Group W's activity (George Deacon, Norman Barber, Fritz Ursell, MS Longuet-Higgins)

- Miles (1957), Phillips (1956): Wave generation by wind
- Cox and Munk (1953/1955): Sea surface slope statistics
- Nonlinear wave resonance (Phillips 1960, Hasselman 1962/1963, Benjamin & Feir 1967, Zakharov 1968)
- Wave breaking (Phillips 1985)