Int. J. Heat Mass Transfer. Vol. 22, pp. 279–287
Pergamon Press Ltd. 1979. Printed in Great Britain

THE TWO-PHASE TURBULENT JET

W. K. MELVILLE† and K. N. C. BRAY
Department of Aeronautics & Astronautics, The University, Southampton, S095NH, England

(*Received* 19 *October* 1977)

Abstract—Turbulent two-phase axisymmetric jets, in which the volume fraction of the secondary phase is much less than unity, are considered. Emphasis is placed on cases in which the mass fraction of particles is of order unity. The available experimental measurements are examined and it is found that physical arguments and dimensional analysis lead to good correlations of the mean fluid velocity and particle mass flux fields in terms of the initial loading of particles. The jet may be simply described with reference to the momentum transfer between the phases. Two main regions exist: a near field in which essentially no momentum has been transferred between the phases, and a far field in which sensibly all the momentum resides in the fluid phase. Exponential and power law functions of the ratio of the mass density of the particles to that of the suspending fluid at the jet orifice are found to correlate much of the data with the corresponding single phase jet. A relationship for the virtual origins of the far field in terms of the integral invariants of the flow is derived and supported by the measurements.

NOMENCLATURE

a, b, define virtual origin of asymptotic far field of momentum and partical mass flux fields;
d, particle diameter;
f, g, h, j, functions defined by similarity solutions of the jet [see equations (3.3) and (3.4)];
m_p, mass flow rate of particles;
r, radial coordinates;
t_e, time scale of energetic fluid turbulence;
t_*, response time of a single particle;
u, velocity scale of energetic turbulence;
v_t, terminal velocity of particles;
x, axial coordinates;
A, B, C, E, F, K, universal constants of similarity solutions [see equations (3.3)–(3.5) and (3.12)];
D, orifice diameter;
G, mean particle mass flux;
M_T, momentum flow rate of particles plus fluid;
M_f, initial fluid momentum flow rate;
Q_0, dimensionless particle response time [see equation (3.1)];
Re_0, jet Reynolds number (see Section 2);
Sc_p, particle field Schmidt number;
T_0, time scale of jet;
U, fluid mean velocity.

Greek symbols
δ, fluid mean velocity half radius;
δ_g, particle mean mass flux half radius;
ν, kinematic viscosity of the suspending fluid;
ρ, density of the suspending fluid;
ρ_s, density of the particle material;
ρ_p, density of the particle field;
χ, ρ_p/ρ.

Subscripts
0, value at jet exit;
m, value at jet axis.

Superscript
0, value in corresponding clean jet.

†Present address: Institute of Geophysics and Planetary Physics, University of California, San Diego, U.S.A.

1. INTRODUCTION

A NUMBER of natural and industrial flows may be considered as two-phase systems. In particular, combusting flows often contain a second condensed phase in the form of fuel, reaction products, or both. In such flows the turbulent mass, momentum and energy transfer processes between the phases may strongly influence the overall efficiency of the combustion. These processes are poorly understood and there is a need for the investigation of simpler prototype flows which emphasise certain features of the more complex situations. One such flow is the two-phase turbulent jet. This is the flow obtained when an inert mixture of particles (or droplets if assumed rigid) and incompressible fluid issues from a nozzle into an unbounded region containing quiescent fluid.

Unlike flows with polymer additives those of most concern in this investigation are such that the secondary phase may be treated as a passive contaminant if its volume and mass fraction are much less than unity. The main topics of concern in these cases are the investigation of the particle velocity in relation to the suspending fluid velocity field, and the transport of the particle mass. As the volume and/or mass fraction of the secondary phase increases so too does the effect of the particles on the primary fluid flow. The particles may make significant contributions to the mass, momentum and energy balances of the mixture, providing additional means of storing and transferring momentum and energy, apart from those already present in the corresponding single phase (or "clean") flow. These processes are not well understood, a situation which, as noted by Owen [1], is due in part to the extensive range of values the interacting variables may take. In an extensive survey Hinze [2] also considered the numerous modes of turbulent particle–fluid interaction.

Despite these difficulties many authors have attempted to consider some aspects of the momentum and energy balances. From a linearised analysis of the equation of motion of a single particle in a homogeneous isotropic flow Kuchanov and Levich [3] concluded that the additional energy dissipation due to the particles' velocity lag may become comparable to the usual viscous dissipation if the ratio of the mass density of the particle cloud to that of the suspending fluid, ρ_p/ρ, is of order unity. Owen [1] obtained essentially the same result by considering the response of a particle having a relaxation time t_*, to a turbulent velocity field having a characteristic time t_e. The assumption of local (energy) equilibrium led to the result that the characteristic velocity of the turbulence, u, was modified by the particles such that,

$$
\begin{aligned}
&u(\rho_p)/u(\rho_p = 0) \\
&\quad \sim [1+\rho_p/\rho]^{-1/2}, \quad t_* \ll t_e \\
&\quad \sim [1+(\rho_p/\rho)(t_e/t_*)]^{-1/2}, \quad t_* \gtrsim t_e.
\end{aligned} \tag{1.1}
$$

Owen went on to discuss the two-phase jet and argued that the force on the fluid due to the particles resulted from the migration of the particles across the mean rate of strain of the mean velocity field. However, this specification of the force is incomplete as other significant contributions to the particles' acceleration are present (see Melville and Bray [4]). By using mixing length arguments Abramovich and Girshovich [5] arrived at an averaged Schmidt number for particle transport dependent on the relative particle mass concentration, ρ_p/ρ.

While studies such as these present convincing arguments for the importance of a number of parameters, in particular (ρ_p/ρ) and (t_*/t_e) they are fragmentary, offering little grasp of the development of the gross features of the development of the two-phase jet. Experience in fluid mechanics has shown that dimensional analysis, correlation of data and simple physical arguments are often useful precursors of more detailed studies. In this work we have followed this course in examining the experimental measurements of the two-phase jet. We are primarily concerned with flows in which the particle mass fraction is significant.

2. THE EXPERIMENTAL MEASUREMENTS

The principal parameters of the experiments reported in the literature are summarised in Table 1.

Table 1. Summary of the characteristic parameters of reported experiments on two-phase turbulent jets

Author	Configuration	Suspension	Particle loading ρ_{p0}/ρ	Particle size d μm	Particle time constant t_* s	Jet exit time constant T_0 s	t_*/T_0	Reynolds number $Re_0 = U_0D/\nu$
Goldschmidt and Eskinazi [8]	Plane	oil/air	10^{-4}	2	0.3×10^{-4}	2×10^{-3}	1.5×10^{-2}	1.6×10^{-4}
Singamsetti [9]	A/S	sand/water	5×10^{-3}	68–460	0.6–8.10^{-3}	10^{-3}	0.6–8	3.9×10^{4}
Householder and Goldschmidt [10]	Plane	DBP/air	$<O(10^{-4})$	6–23	$O(10^{-4}$–$10^{-3})$	$O(10^{-3})$	$O(10^{-1}$–$1)$	$>3.8\times10^{4}$
Hetsroni and Sokolov [6]	A/S	oil/air	2–8×10^{-3}	13	0.6×10^{-3}	0.5×10^{-3}	1.2	8×10^{4}
Laats [13]	A/S	shale/air	0–1	20–60	$O(10^{-3})$	0.5–2.3×10^{-3}	$O(1)$	$>2\times10^{4}$
Laats and Frishman [15, 16]	A/S	corundum/air	0–1.4	17–80	$O(10^{-3}$–$10^{-2})$	$O(10^{-3})$	$O(1$–$10)$	$O(10^{5})$
Ivanov, Laats and Frishman [14]	A/S	corundum/air	0–3	20–60	$O(10^{-3})$	$O(10^{-3})$	$O(1)$	$O(10^{5})$

The particle loading $\chi_0 = \rho_{p0}/\rho$ is the ratio of the mass density of the particle cloud at the nozzle, to the density of the incompressible suspending fluid. For our purposes, where we are mainly concerned with orders of magnitude, it is sufficient to represent the particle response time t_* by the Stokesian time constant, $t_* = (d^2/36\nu)(2\rho_s/\rho+1)$, where d is the diameter of the particle, ρ_s the density of the particle material, and ν the kinematic viscosity of the fluid. Owen's work stressed the importance of the ratio t_*/t_e, where t_e is the time scale of the energetic eddies of the fluid turbulence. In the table we give the characteristic time scale of the flow as $T_0 = D/U_0$, where U_0 is the fluid velocity at the nozzle and D is the nozzle diameter. In the clean axisymmetric jet $t_e \sim O(10^{-1})T_0(x/D)^2$ and we anticipate that in the flows considered here, where χ_0 is at most of order unity, the same relationship will provide an acceptable order of magnitude estimate. In fact we expect that all the relevant fluid flow scales will be of the same order of magnitude as those at the same station in the corresponding clean flow.†

Of the measurements in lightly loaded jets ($\chi_0 < 10^{-2}$) only those of Hetsroni and Sokolov [6] showed a significant change in the fluid mean velocity field from that of the clean jet ($\chi_0 = 0$). However, their results should be viewed with caution as even their clean jet measurements differed considerably from those of a number of other workers (cf. Harsha [7], Fig. 9.5). In addition, we have estimated that due to the frequency of droplet impact their hot wire anemometer was contaminated much of the time and not likely to give a reliable response to the fluid velocity. Notwithstanding Hetsroni and Sokolov's measurements, we may conclude that for light loadings ($\chi_0 \ll 1$) the fluid mean velocity field is sensibly unchanged from that of the clean jet.

All the measured particle mean mass flux profiles attained a self-similar form. Goldschmidt and Eskinazi, with the smallest particles listed in Table 1, [8], found that the mean velocity and mass flux fields could be related by a Schmidt number (Sc_p) of 1.1. Singamsetti [9] found Sc_p to be independent of the streamwise position, ranging from 0.85 for the smallest particles to 0.69 for the largest. The decrease of Sc_p corresponds to a broadening of the particle concentration profile. Householder and Goldschmidt [10] also found a decrease in Sc_p, from 0.42 to 0.30, with increasing particle size. These reported values of Sc_p do not differ greatly from the corresponding values of 0.7 (axisymmetric) and 0.5 (plane) found for passive gaseous contaminants (Launder and Spalding [11]). One feature of their results [10] which deserves comment is that values of G_m/G_0, the ratio of axial particle mass flux to the value at the orifice, were shown to be greater than unity just downstream of the orifice. In reviewing these measurements Goldschmidt *et al.* [12] reported some uncertainty in the measured values of G_0; however, it is unlikely that this would account for the values of G_m/G_0 of $O(10)$ which were presented.

To our knowledge the only measurements of heavily loaded jets reported in the literature are those of Laats and his colleagues. All of these experiments were with solid particles in a suspending air stream. In each case the particles were accelerated to the air velocity at the jet orifice. Laats [13] reported that the ratio of the fluid mean velocity on the axis to that at the orifice, U_m/U_0, showed no dependence on $Re_0 = U_0D/\nu$ or T_0, depending only on x/D for a given particle loading. An increase in χ_0 led to a decrease in the rate of decay of U_m/U_0 and a decrease in the velocity half-radius δ. Over the range measured ($0 \leqslant r \leqslant 1.5\delta$, $x/D \geqslant 5$) the mean velocity profiles, U/U_m, were found to have the same self-similar form as the clean axisymmetric jet.

The experiments of Ivanov *et al.* [14] were preliminary to those of Laats and Frishman [15, 16] who found that the particle mass flux profiles attained similarity at $x/D \simeq 10$, and were well fitted by the profile

$$G/G_m = \exp[-0.7(r/\delta_g)^{4/3}]$$

where G_m is the axis value, and δ_g the half-radius. The fluid velocity profiles were not strictly self-similar but approached the self-similar form of the clean jet profiles on moving downstream. However, all the velocity profiles presented are within the bounds of the measurements of the clean jet profiles made with comparable techniques (Hinze *et al.* [17]). The axial velocity was found to decay more slowly with increasing χ_0. The effect of a change in particle size is not so clear. For $\chi_0 = 0.3$, the rate of decay of U_m was found to increase as d went from 32 to 72 μm, but the opposite trend was apparent for $\chi_0 = 0.56$. The measured values of G_m/G_{0m} (where G_{0m} is the centreline particle mass flux at the nozzle exit) are also of interest, with some showing maxima ($G_m/G_{0m} = 1.1$–1.2) just downstream of the jet orifice. The effect, which is more pronounced with the smaller particles, was attributed by Laats and Frishman to a radial transport of the particles by Magnus forces.

3. DIMENSIONAL ANALYSIS OF MEAN FLOW VARIABLES

3.1. *Similarity solutions*

We consider an axisymmetric two-phase jet issuing from an orifice with a uniform velocity U_0 and secondary phase density ρ_{p0}. The particles have a characteristic diameter d, which is much less than the smallest scale of the fluid velocity field. The secondary phase volume fraction is much less than unity. If the only significant form of particle–fluid interaction is a linear viscous drag the dynamics of a particle may be represented by a characteristic response time t_*.

From dimensional considerations it follows that the fluid mean velocity $U(x, r)$ must be given by an equation of the form

$$U/U_0 = f(x/D, r/D, Re_0, \chi_0, Q_0) \qquad (3.1)$$

where

$$Q_0 = t_*/T_0.$$

The experiments cited above show the mean velocity field to be sensibly independent of the Reynolds number, Re_0, for sufficiently large Re_0. In addition, we expect that if t_* is small, much smaller than the time characteristic of the energetic turbulence scales, the mean velocity field will be independent of the

---

†By "corresponding clean flow" we mean that single phase flow which is obtained by eliminating the particles; all other boundary conditions pertaining to the fluid remaining the same.

time parameter Q_0. With these restrictions we have that

$$U_m/U_0 = f(x/D, \chi_0) \tag{3.2}$$

where U_m is the velocity on the axis, $U(x, 0)$.

On the basis of their measurements Laats and Frishman [16] suggested that similarity solutions of the mean velocity and particle mass flux fields may be good approximations over significant sections of the jet. Such solutions require the magnitude of the profiles to vary as some power of x [18]. With the constraint that in the limit as χ_0 tends to zero the jet behaves as one containing a passive contaminant, the similarity solutions consistent with equation (3.2) is

$$U_m/U_0 = A\left(\frac{D}{x-a}\right)f(\chi_0), \quad f(0) = 1 \tag{3.3a}$$

$$\delta = B(x-a)g(\chi_0), \quad g(0) = 1. \tag{3.3b}$$

In a similar fashion it may be shown that the separation of variables required by the similarity solution and the physical constraint that as χ_0 tends to zero the particles behave as a passive contaminant lead to the following relationships for the mass flux field:

$$G_m/\rho_{p0}U_0 = C\left(\frac{D}{x-b}\right)^2 h(\chi_0), \quad h(0) = 1 \tag{3.4a}$$

$$\delta_g = E(x-b)j(\chi_0), \quad j(0) = 1. \tag{3.4b}$$

A, B, C and E are universal constants, and $x = a,b$ are the virtual origin of the fluid momentum and particle mass respectively.

3.2. *Far field solutions*

The mixing of the jet with the entrained ambient primary fluid results in the mean particle density decreasing downstream.

For the response times being considered the mean velocity of the particles and fluid are sensibly equal. Thus the momentum flux of the particles decreases, the momentum being transferred to the suspending fluid. In the far field, where $\rho_p/\rho \ll 1$, essentially all the momentum is carried by the fluid.

From the evidence presented in Section 2 we concluded that jets having a low initial density ratio, χ_0, are dynamically unaffected by the particles. The far field too is independent of the particle dynamics and will develop as a single phase jet having a momentum flow rate M_T, where M_T is the sum of the initial particle and fluid momentum flow rates. Such a jet becomes self preserving and may be described by the following equations due to Corrsin [19]:

$$U_m = K\left[\frac{M_T}{2\pi\rho}\right]^{1/2}\frac{1}{(x-a)}, \quad K = \text{const.} \tag{3.5a}$$

$$\delta = B(x-a). \tag{3.5b}$$

For the corresponding clean jet having a virtual origin at $x = 0$, the axis velocity U_m^0 is given by

$$U_m^0 = K\left[\frac{M_f}{2\pi\rho}\right]^{1/2}\frac{1}{x} \tag{3.6}$$

where M_f is the fluid momentum flow rate at the jet orifice. In the far field, as $x \to \infty$,

$$U_m/U_m^0 \to [M_T/M_f]^{1/2}; \tag{3.7}$$

but for the uniform flow at the orifice

$$M_T/M_f = 1+\chi_0. \tag{3.8}$$

It follows from equations (3.3), (3.7) and (3.8) that in the far field

$$f(\chi_0) = (1+\chi_0)^{1/2} \tag{3.9a}$$

$$g(\chi_0) = 1. \tag{3.9b}$$

Unlike the fluid momentum, the particle mass flux is independent of the streamwise co-ordinate, and the far field is described by a change in the virtual origin, b, and $h(\chi_0)$ and $j(\chi_0)$ are found to be independent of χ_0. Thus:

$$h(\chi_0) = j(\chi_0) = 1. \tag{3.10}$$

Immediately below we consider the dependence of the virtual origin on the loading χ_0.

3.3. *The virtual origin of the far field*

In the far field the averaged profiles can be functions of only ρ, v, x, r, t_*, M_T and m_p where m_p is the particle mass flow rate. It follows from dimensional reasoning that

$$\delta = Bxfn\left[\frac{m_p}{M_T}\left(\frac{M_T}{\rho}\right)^{1/2}\frac{1}{x}, \frac{t_*}{x^2}\left(\frac{M_T}{\rho}\right)^{1/2}, \left(\frac{M_T}{\rho}\right)^{1/2}\frac{1}{v}\right]. \tag{3.11}$$

Neglecting Reynolds number and response time effects (3.11) becomes

$$\delta = Bxfn\left[\frac{m_p}{M_T}\left(\frac{M_T}{\rho}\right)^{1/2}\frac{1}{x}\right]$$

$$= Bx(1-a/x), \text{ from (3.5b).}$$

It follows immediately that the virtual origin a is given by

$$a = Fm_p/(\rho M_T)^{1/2} \tag{3.12}$$

where F is a constant. For uniform jet orifice conditions

$$a = F\frac{(\pi)^{1/2}}{2}\frac{\chi_0}{(1+\chi_0)^{1/2}}D. \tag{3.13}$$

The same argument shows that b, the virtual origin of the particle mass flux field, is also proportional to $\chi_0/(1+\chi_0)^{1/2}$.

4. CORRELATION OF THE EXPERIMENTS

We wish to compare our predictions of Section 3 with the experiments of Laats [13] and Laats and Frishman [15, 16]. First we must determine whether the simplifying assumptions made in Section 3.1. are applicable to the experimental conditions. We have

Table 2. Dimensionless ratios characterising the particle–fluid interaction in the experiments of Laats [13] and Laats and Frishman [15, 16]

	d/η	t_*/t_e	Re	v_t/u
Laats [13]	$O(10^{-1})$	$O(10^{-1})$	$O(1)$	$O(10^{-2})$
Laats and Frishman [15, 16]	$O(10^{-1}\text{–}1)$	$O(10^{-1}\text{–}1)$	$O(1)$	$O(10^{-2}\text{–}10^{-1})$

made magnitude estimates of the relevant quantities, and they are presented in Table 2. All the estimates are for $x/D \sim O(10)$, and are based on the assumption that the scales are of the same order of magnitude as those in the corresponding clean flow. The first column of the table shows the ratio of the particle diameter to the Kolmogorov length scale. This ratio is at most of order unity so we expect direct particle size effects to be negligible. The ratios of t_*/t_e are shown in the next column, and are in the range $O(10^{-1}-1)$, so we might expect that in some of the experiments the dependence on t_* was negligible. In both cases we estimate that the Reynolds number Re based on the particle diameter and the velocity lag is at most of order unity, so that characterising the particle response by t_* is acceptable. The ratio of the particle terminal velocity and turbulence velocity scale, v_t/u, is in both cases much less than unity, and from Owen [1] we estimate that the work done in sustaining the particles against gravity is negligible. Gravitational effects will become more important downstream with v_t/u increasing due to the decay of the turbulence. The maxima in the axial mass flux measurements, as mentioned above, suggest that shear and Magnus forces may not be negligible, but these effects appear to be restricted to the region just downstream of the nozzle. One point which should be mentioned is that both Laats [13] and Laats and Frishman [15, 16] reported nonuniformity in their initial profiles of fluid mean velocity and particle mass flux, and normalised their data with respect to the maximum (centreline) values U_{0m} and G_{0m} at the nozzle exit. Laats and Frishman [16] indicated that the profile changes in the range $0.1 < \chi_0 < 0.7$ were slight, so the effect on the ratios U_{0m}/U_0 and G_{0m}/G_0 were slight and the absolute values should be absorbed in the correlations presented below.

Laats [13] measurements of U_m/U_{0m} are well correlated by the relationship

$$U_m/U_{0m} = A\frac{D}{x}\,\mathrm{e}^{0.69\chi_0} \tag{4.1}$$

over the entire axial range of his measurements (Fig. 1). In the near field the half radius δ is well correlated by

$$\delta = Bx\,\mathrm{e}^{-0.69\chi_0}. \tag{4.2}$$

Due to the self similarity of the velocity profiles the momentum flow rate of fluid is proportional to $(U_m\delta)^2$, which is approximately constant in the region in which both (4.1) and (4.2) are fairly successful in correlating the measurements; $x/D\,\mathrm{e}^{-0.69\chi_0} \lesssim 7$ say. In this region there has been essentially no momentum transfer between the phases, but the variation of experimental data for δ with χ_0 indicates a systematic deviation from (4.2) with increasing x corresponding to a transfer of momentum from the particles to the fluid. In Fig. 2 we have replotted the results of Laats and Frishman for the 32 μm particles using the exponential scaling.

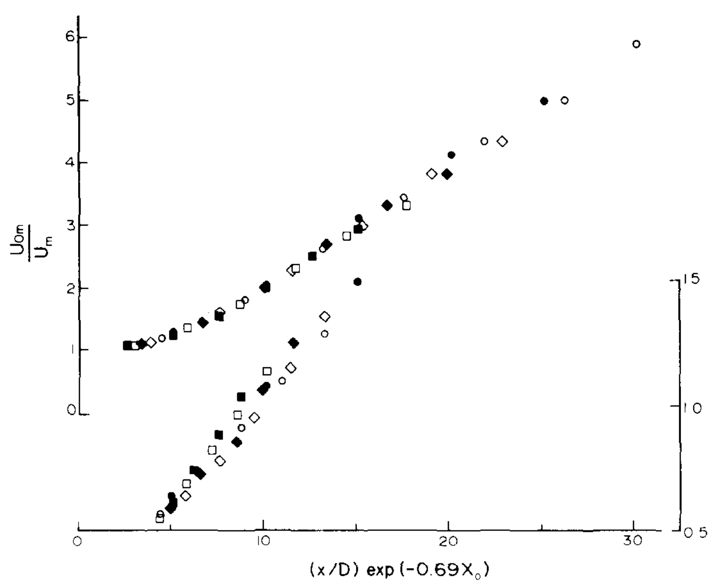

FIG. 1. Exponential correlation of Laats [13] measurements of the fluid mean axial velocities and velocity half-radii. $\chi_0 = 0$ ●, 0.2 ○, 0.4 ◇, 0.6 ◆, 0.8 □, 1.0 ■.

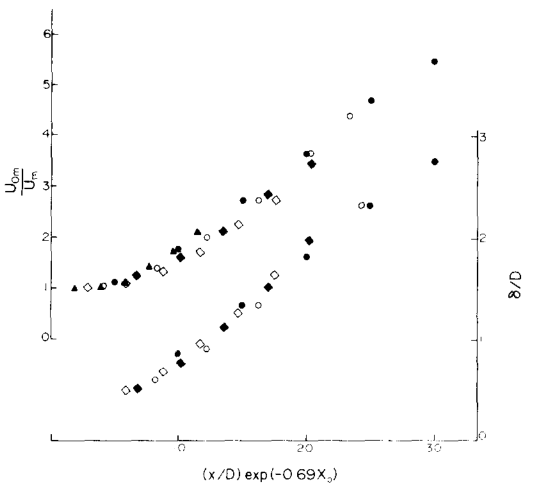

FIG. 2. Exponential correlation of Laats and Frishman's [15, 16] measurements of the fluid mean axial velocities and velocity half-radii for the particle size $d = 32\,\mu$m. $\chi_0 = 0$ ●, 0.3 ○, 0.56 ◆, 0.77 ◇, 1.4 ▲.

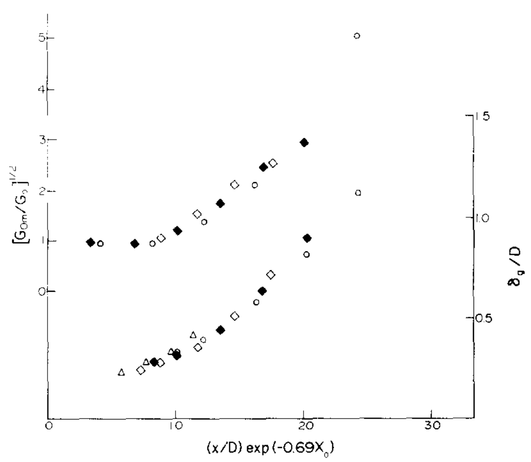

FIG. 4. Exponential correlation of Laats and Frishman's [15, 16] measurements of the mean particle mass flux field for the particle size $d = 32\,\mu$m. $\chi_0 = 0.3$ ○, 0.56 ◆, 0.77 ◇.

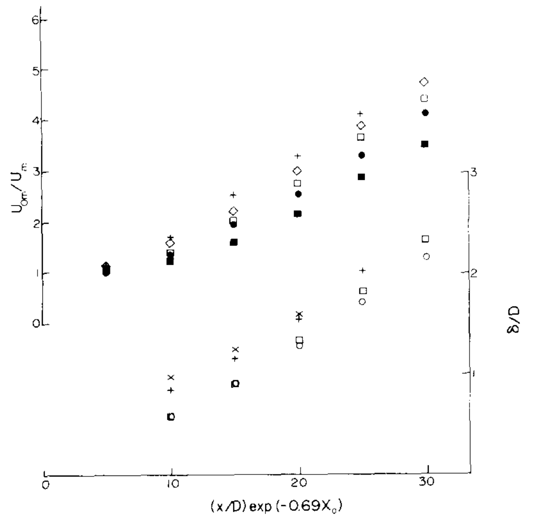

FIG. 3. The dependence of the mean fluid velocity field on the particle size. $\chi_0 = 0.3$; $d = 17$ ○, 32 □, 49 ◇, 72 +, 80 ×. $\chi_0 = 0.56$; $d = 17$ ●, 32 ■. For χ_0 fixed the main effect of the change in particle size is to shift the curves axially (Laats and Frishman [15, 16]).

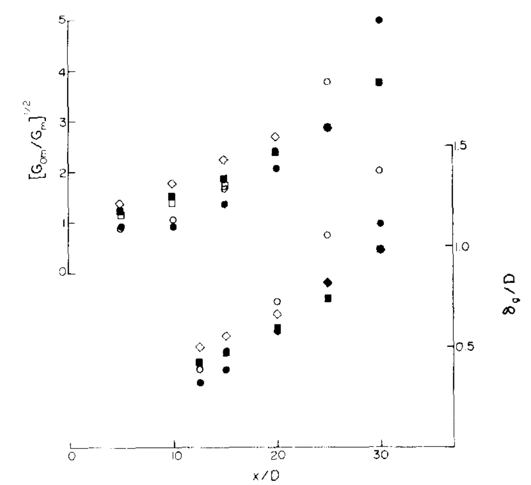

FIG. 5. Measurements of the mean particle mass flux field for $\chi_0 = 0.3$ and varying particle size. $d = 17$ ○, 32 ●, 49 □, 72 □, 80 ◇ (after Laats and Frishman [15, 16]). Note the slight minima in $(G_{0m}/G_m)^{1/2}$ for $d = 17, 32$.

The correlation of U_{0m}/U_m is not as good as that of Fig. 1, but the scatter is no greater than that found in the corresponding measurements of clean jets [7].

Figure 3 displays the effects of varying the particle size, d. For $\chi_0 = 0.3$, increasing d in the range 32 to 72 µm leads to an increase in U_{0m}/U_m, while the opposite trend is evident for $\chi_0 = 0.56$, as d goes from 17 to 32 µm. It is evident that these measurements are not independent of t_*. This is not surprising as t_*/t_e ranged up to $O(1)$ over these particle sizes. Unfortunately Laats and Frishman's reporting of the experiments does not indicate whether T_0 was varied, so we are unable to seek any dependence on t_*/T_0.

The particle field mean mass flux variables for $d = 32\,\mu$m are replotted in Fig. 4 using the exponential scaling. The points are shown to correlate well and with the exception of one point are linear in x for $(x/D)e^{-0.69\chi_0} \gtrsim 10$, as required by the similarity solution. Figure 5 shows the corresponding measurements for a fixed loading, $\chi_0 = 0.3$, with d varying from 17 to 80 µm. There is no obvious order associated with changes in d. This is due in part to the behaviour of the flows containing the smaller particles (17 and 32 µm) which display maxima in G_m/G_{0m} in the neighbourhood of the nozzle.

The experimental results of Laats and Laats and Frishman show (at least for the lighter loadings) that the velocity half-radii curves tend to a slope equal to that of the corresponding clean jet as predicted in equation (3.9b). It remains to determine whether the virtual origin $(x = a)$ defined in that equation

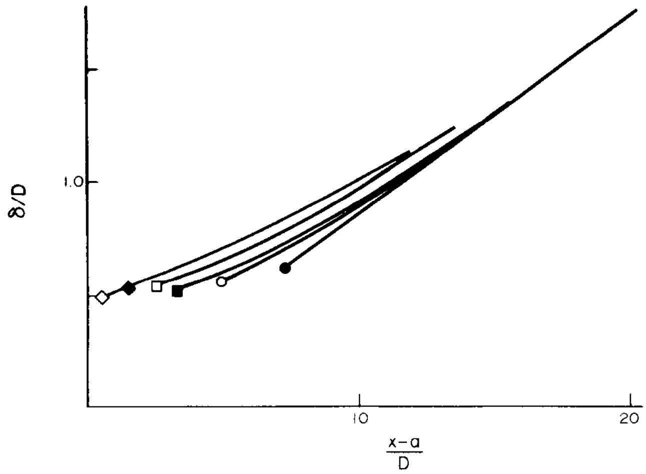

FIG. 6. Laats [13] velocity half radii replotted relative to the far field origin ($x = a$) given by equation (3.13). $\chi_0 = 0$ ●, 0.2 ○, 0.4 ■, 0.6 □, 0.8 ◆, 1.0 ◇.

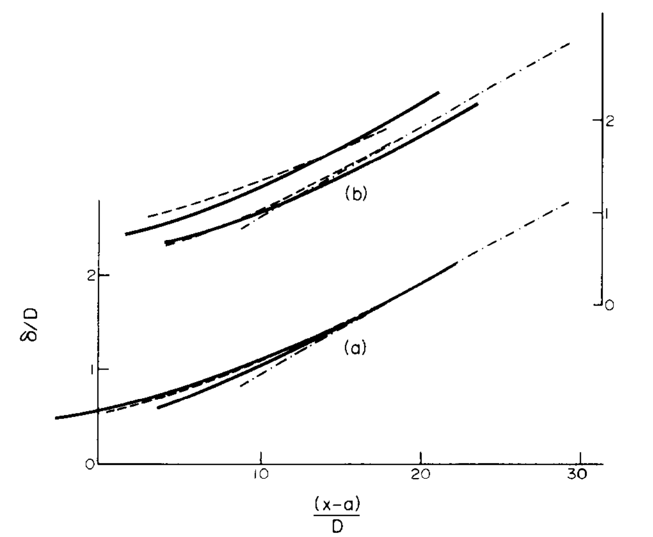

FIG. 7. Laats and Frishman's [15, 16] velocity half-radii plotted relative to the far field origin ($x = a$) given by equation (3.13). (a) $d = 32$: $\chi_0 = 0$ —·—; 0.3 ——; 0.56 - - -; 0.77 ——. (b) $\chi_0 = 0.3$: $d = 17$ ——; 32 - - -; 72 ——; 80 - - -.

follows the behaviour predicted in equation (3.13). As only the curves for the lighter loadings had achieved the asymptotic state we were unable to empirically determine a for each loading so we have used the following method to display the results. The empirical value of a determined for the lightest loading in each set of experiments was fitted to equation (3.13) to give a value for the "constant" F. This constant was then used in equation (3.13) to compute values of a for the other loadings. The experimental curves were then plotted as δ/D vs $(x-a)/D$. From Laats results for $\chi_0 = 0.2$ the computed value of F was found to be 15.2. The replotted half-radii are shown in Fig. 6; if the correlation is successful the curves will all approach that for the unseeded jet at large axial distances. The correlation for $\chi_0 = 0.4$ is good but the heavier loaded jets have not attained the asymptotic state. The computed value of F from Laats and Frishman's measurements for $\chi_0 = 0.3 (d = 32\,\mu\text{m})$ was found to be 22.3. The difference in the two values for F may be caused by a number of factors including nozzle design and particle size effects. The data for $d = 32$ are shown in Fig. 7(a) and correlate very well. The effects of particle size are apparent in 7(b), where it is clear that the virtual origin may depend to a significant extent on the particle size. The measurements of δ_g were not extensive enough to allow a correlation of the far field virtual origin of the mass flux field.

The experimental measurements have not been continued far enough downstream for the asymptotic behaviour of U_{0m}/U_m to appear clearly. For the lighter loadings it is possible that the differences between the power-law and exponential scaling are within the bounds of experimental error. For example, for $\chi_0 = 0.2$, $e^{0.69\chi_0}/(1+\chi_0)^{1/2} = 1.05$. The heavier loaded jets, for which the difference would be discernible [e.g. $e^{0.69\chi_0}/(1+\chi_0)^{1/2} = 1.41$ for $\chi_0 = 1.0$], have not reached the asymptotic state within the streamwise extent of the measurements.

5. DISCUSSION

The analysis and experimental evidence show that the gross features of the development of the mean velocity field of the fluid may be described with reference to the momentum transfer between the phases. Two main regions exist: a near field in which essentially no net momentum transfer has occurred, and a far field in which sensibly all the momentum of the particles has been transferred to the suspending fluid. Correlation of the experimental data shows that the former region may be related to the corresponding clean jet by an exponential function of the initial particle loading; while physical arguments show that a power law describes the latter. The power law does not successfully correlate the near field data. In the far field region the dynamics of the primary fluid are sensibly equivalent to a clean jet issuing from a virtual origin, the position of which is dependent on the initial particle loading and the particle size.

For a constant particle size we have found good agreement with the predicted dependence of the virtual origin on χ_0. The dependence on d is less clear. However, we expect that one of the principal effects of varying d is to change the rate at which momentum is transferred from the particles to the fluid, with a consequent change in the position of the virtual origin of the far field: a lower rate of transfer leading to a downstream displacement.

The particle mass flux field is evidently strongly influenced by particle size effects, with both dilute and heavily loaded flows showing extrema in the mass flux fields in the region immediately downstream of the nozzle. Laats and Frishman suggest that this is due to Magnus forces transporting particles towards the axis. The effect is more pronounced with the finer particles but according to Hinze [2] this force should be negligible for smaller particles.

It must be emphasised that the correlations presented here are based on one group's measurements which, due to the difficulties of data acquisition in two-phase systems, are perhaps not as reliable as those made in less testing flows. While we have found the measurements of Laats and his colleagues to be self consistent with regard to the simple checks of mass and momentum conservation, there is a clear need for independent experiments. Confirmation of the exponential scaling would be desirable as we have been unable to find a simple physical argument leading to this result.

The present work has given a simple overview of the gross features of the development of the two-phase jet, with particular regard to effects of the particle loading χ_0. In a subsequent paper [4] we examine in more detail the turbulent particle–fluid interaction, using a set of Reynolds averaged equations and a first order closure scheme which accounts for both particle concentration and size effects.

Acknowledgements—We wish to thank Professor G. N. Abramovich for drawing our attention to some of the work of Laats and Frishman. One of us (W.K.M.) acknowledges the support of a Hawker Siddeley Fellowship.

REFERENCES

1. P. R. Owen, Pneumatic transport, *J. Fluid Mech.* **39**, 407 (1969).
2. J. O. Hinze, Turbulent fluid and particle interaction, *Prog. Heat Mass Transfer*, Vol. 6, edited by G. Hetsroni. Pergamon Press, Oxford (1972).
3. S. I. Kuchanov and V. G. Levich, Energy dissipation in a turbulent gas containing suspended particles, *Soviet Physics-Doklady* **12**, 549 (1967).
4. W. K. Melville and K. N. C. Bray, A model of the two-phase turbulent jet, *Int. J. Heat Mass Transfer*. To be published.
5. G. N. Abramovich and T. A. Girshovich, Diffusion of heavy particles in turbulent gas streams, *Soviet Physics-Doklady* **18**, 587 (1974).
6. G. Hetsroni and M. Sokolov, Distribution of mass, velocity and intensity of turbulence in a two-phase turbulent jet, ASME Appl. Mech. Divn Paper No. 70-WA/APM-45 (1970).
7. P. T. Harsha, Free turbulent mixing: A critical evaluation of theory and experiment, Arnold Engineering Development Center, AEDC-TR-71-36 (1971).
8. V. Goldschmidt and S. Eskinazi, Two-phase turbulent flow in a plane jet, *J. Appl. Mech.* **33**, 735 (1966).
9. S. R. Singamsetti, Diffusion of sediment in a submerged jet, *Proc. Am. Soc. Civ. Engrs* **HY2**, 153 (1966).
10. M. K. Householder and V. W. Goldschmidt, Turbulent diffusion of small particles in a two-dimensional free jet, Tech. Rep. FMTR-68-3, School of Mech. Engng, Purdue University (1968).
11. B. E. Launder and D. G. Spalding, *Mathematical Models of Turbulence*. Academic Press, London (1972).
12. V. W. Goldschmidt, M. K. Householder, G. Akmadi and S. C. Chuang, Turbulent diffusion of small particles suspended in turbulent jets, *Progress in Heat Mass Transfer*, Vol. 6, edited by G. Hetsroni. Pergamon Press, Oxford (1972).
13. M. K. Laats, Experimental study of the dynamics of an air-dust jet, *Inzh. Fiz. Zh.* **10**, 1 (1966).
14. Y. V. Ivanov, M. K. Laats and F. A. Frishman, Dispersion of a heavy admixture in a two-phase jet, *Inzh. Fiz. Zh.* **18**, 538 (1970).
15. M. K. Laats and F. A. Frishman, Scattering of an inert admixture of different grain size in a two-phase axisymmetric jet, *Heat Transfer–Soviet Res.* **2**, 7–11 (1970).
16. M. K. Laats and F. A. Frishman, Assumptions used in calculating the two-phase jet, *Fluid Dynamics* **5**, 333 (1970).
17. J. O. Hinze and B. G. van der HeggeZijnen, Transfer of heat and matter in a symmetrical jet, *Appl. Scient. Res.* **1A**, 435 (1949).
18. G. K. Batchelor, Heat convection and buoyancy effects in fluids, *Q. Jl R. Met. Soc.* **80**, 339 (1954).
19. S. Corrsin, Comment on "Diffusion of submerged jets", *Trans. Am. Soc. Civ. Engrs* **115**, 676 (1950).

LE JET TURBULENT DIPHASIQUE

Résumé—On considère des jets turbulents, axisymétriques et diphasiques dans lesquels la fraction volumique de la phase secondaire est très faible. On porte attention aux cas pour lesquels la fraction massique des particules est de l'ordre de l'unité. On examine les mesures expérimentales disponibles et on trouve que les arguments physiques et l'analyse dimensionnelle conduisent à des formules liant la vitesse moyenne du fluide et le flux massique de particules à la charge initiale de particules. Le jet peut être simplement décrit à partir du transfert de quantité de mouvement entre les phases. Deux régions principales existent: un champ proche dans lequel aucun transfert de quantité de mouvement n'opère entre les deux phases et un champ lointain dans lequel toute la quantité de mouvement est dans le fluide. Des fonctions, exponentielles et en puissance, du rapport de la masse volumique des particules à celle du fluide à l'orifice du jet représentent la plupart des résultats avec le jet correspondant à une seule phase. On dérive une formule pour les origines virtuelles du champ lointain en fonction des invariants intégraux de l'écoulement, laquelle s'accorde avec les mesures.

DER ZWEIPHASIGE TURBULENTE STRAHL

Zusammenfassung—Turbulente zweiphasige achsensymmetrische Strahlströmungen, in denen der Volumenanteil der zweiten Phase sehr viel kleiner als eins ist, werden detrachtet. Besonders betont werden die Fälle, in denen das Massenverhältnis der Partikel von der Größenordnung eins ist. Die verfügbaren Versuchsdaten werden ausgewertet, und man findet, daß physikalische Gründe und Dimensionsanalyse zu guten Korrelationen zwischen der mittleren Fluidgeschwindigkeit und den Massenstromfeldern der Partikel in Abhängigkeit von der Anfangsbeladung führen. Der Strahl läßt sich unter Berücksichtigung des Impulsaustausches zwischen den Phasen einfach beschreiben. Es gibt zwei Hauptgebiete: einen Nahbereich, in dem im wesentlichen noch kein Impulsaustausch zwischen den Phasen stattgefunden hat, und einen Fernbereich, in dem sich der gesamte Impuls offensichtlich in der Flüssigphase befindet. Es wird festgestellt, daß bei Exponential- und Potenzfunktionen des Verhältnisses von Massendichte der Partikel zur Massendichte der sie tragenden Flüssigkeit am Düsenaustritt viele der Meßdaten mit dem entsprechenden Einphasenstrahl korrelieren. Eine Beziehung für den virtuellen Ursprung des Fernbereiches in Abhängigkeit von mittleren Kennwerten der Strömung wird abgeleitet und durch Messungen bestätigt.

ДВУХФАЗНАЯ ТУРБУЛЕНТНАЯ СТРУЯ

Аннотация — Рассматриваются турбулентные двухфазные осесимметричные струи, объемная доля вторичной фазы которых намного меньше единицы. Особое внимание обращено на случаи, когда массовая доля частиц близка к единице. Проведен анализ имеющихся экспериментальных данных и найдено, что с помощью физических соображений и анализа разномерностей можно получить хорошие обобщенные соотношения для полей средней скорости жидкости и массовых потоков частиц, выраженных через их начальное содержание. Перенос импульса между фазами в струе может быть описан с помощью простых допущений, т. е. предположения о том, что в струе существуют две основные области: ближнее поле, в котором, в основном, отсутствует перенос импульса между фазами и дальнее поле, в котором весь импульс сохраняется в жидкой фазе. Найдено, что с помощью экспоненциальных и степенных законов для отношения плотности массы частиц к плотности взвешивающей жидкости на выходе струи можно обобщить большинство данных по однофазным струям. Выведено соотношение для определения начала возникновения дальнего поля, выраженное через интегральные инварианты, и получено его экспериментальное подтверждение.