Energy Dissipation by Breaking Waves

W. KENDALL MELVILLE

Scripps Institution of Oceanography, University of California, San Diego, La Jolla, California

(Manuscript received 7 December 1992, in final form 10 October 1993)

ABSTRACT

Recent field measurements by Agrawal et al. have provided evidence of a shallow surface mixed layer in which the rate of dissipation due to turbulence is one to two orders of magnitude greater than that in a comparable turbulent boundary layer over a rigid wall. It is shown that predictions by Phillips of the energy lost by breaking surface waves in an equilibrium regime and laboratory measurements by Rapp and Melville of the mixing and turbulence due to breaking together lead to estimates of the enhanced dissipation rate and the thickness of the surface layer consistent with the field measurements. Wave-age-dependent scaling of the dissipation layer is proposed. Laboratory measurements of dissipation rates in both unsteady and quasi-steady breaking waves are examined. It is shown that an appropriately defined dimensionless rate of dissipation in unsteady breaking waves is not constant, but increases with a measure of the wave slope. Differences between dissipation rates in quasi-steady and unsteady breakers are discussed. It is found that measurements of the dissipation rate in unsteady breakers are consistent with independent estimates of the turbulent dissipation. The application of these results to models of dissipation due to breaking and air–sea fluxes is discussed.

1. Introduction

The dynamical coupling between the atmosphere and the ocean is mediated by the surface wave field. Estimates by Mitsuyasu (1985) supported by laboratory measurements of Melville and Rapp (1986) and Rapp and Melville (1990, hereinafter RM) suggest that a large fraction of the momentum flux from the atmosphere to the ocean is initially associated with wave generation, but only a small fraction of that (perhaps 5% or so) is carried by the waves propagating out of the generating region. The difference accounts for the transfer to currents by wave breaking. Breaking waves also limit the height of surface waves, are a source of vorticity and turbulence, and enhance gas transfer by both enhancing surface turbulence and by entraining bubbles. Breaking waves also dissipate surface wave energy, making energy available for mixing the surface layers. It is the dissipation of wave energy that is the subject of this paper.

The evolution of weakly nonlinear surface gravity waves is usually modeled by a radiative transfer equation describing the evolution of the wave action density as a function of wavenumber with input from the wind, nonlinear transfers due to quartic interactions, and dissipation due to breaking (Phillips 1977). The foundations of this model are based on essentially linear kinematics with the dynamical wind input according to the Miles (1957, 1992) model along with empirical correlations of field and laboratory data for wave growth (Plant 1982; Snyder et al. 1981). Nonlinear transfers are computed according to Hasselmann's (1962) "collision integral." For operational wave forecasting a heuristic treatment of the dissipation due to wave breaking is usually based on Hasselmann's (1974) model (see also Komen et al. 1984). While there is not complete agreement on the adequacy of the models for the wind input and the nonlinear transfers, it is fair to say that they are based on rational theories and are, in principle, testable.

The primary inadequacy of the wind–wave models is the dissipation term. For mixed-layer models, and more general modeling of air–sea interaction, this is the most important term because it acts as a source of energy for the water column: energy that is available for generating currents and mixing across the air–sea interface. This includes mixing of both heat and gases. Fluxes across the air–sea interface may be enhanced by the local increase in turbulence associated with breaking, and in the case of gas transfer, by the entrainment of air that breaks up into bubbles.

Both models and field measurements of evolving wave fields often use an implicit multiple-scale approach to the problem. On a local scale, which may be many wavelengths and periods, the wave field is considered to be homogeneous and stationary, whereas on much longer length and timescales it is considered to be evolving. The elements of this approach were considered by Phillips (1985), who proposed an equilibrium range for wavenumbers large compared with

Corresponding author address: Prof. W. K. Melville, Scripps Institution of Oceanography, University of California at San Diego, La Jolla, CA 92093-0213.

the spectral peak, in which the wind input, nonlinear transfers, and dissipation were in local balance. The evolution was considered to be self-similar to the extent that each of the three terms were proportional to one another. As a consequence of these assumptions, Phillips was able to use our better knowledge of the input and nonlinear transfer terms to account for the dissipation. The theory also provided expressions for the spectral rates of action, energy, and momentum loss from the equilibrium range due to wave breaking, and integrated rates across the whole range. Phillips also used the predicted integrated dissipation along with laboratory measurements of dissipation in quasi-steady breaking waves (Duncan 1981) to infer statistical descriptions of breaking based on the length of breaking fronts per unit area of the surface in a specified phase speed interval. These predictions remain to be tested against measurements.

Very recently, Agrawal et al. (1992) have measured enhanced levels of dissipation in the surface layers of a lake. Their measurements support earlier lake and ocean measurements by a number of workers. Working from a platform in Lake Ontario they used optical, acoustical, and electromechanical instruments to measure the dissipation near the surface. Their principal results and measurements by others are shown in Fig. 1, which presents the measured dissipation scaled by $u_{*w}^3/\kappa z$, where u_{*w} is the friction velocity in the water, κ is von Kármán's constant (0.4), and z is the depth below the surface. This is the wall-layer scaling, which for boundary layers over solid surfaces would give a normalized dissipation rate of unity. The depth is scaled by u_{*w}^2/g, which is proportional to the significant wave height for fully developed waves. Figure 1 shows that the measured rates of dissipation were up to 70 times greater than those in a comparable wall layer down to depths of $O(10^5)$ in dimensionless units. The scatter in the dissipation rates, especially as the surface is approached, is undoubtedly due in part to undersampling of the intermittent turbulent field; however, there can be no doubt about the trend of the data. Agrawal et al. suggested that independent estimates of energy fluxes to steep waves at frequencies above the peak of the spectrum were consistent with the enhanced dissipation being due to the breaking of larger waves, which lead to whitecaps.

Estimates of the volumetric rate of dissipation in the surface layer depend not only on energy fluxes to steep waves but also on estimates of the depth over which the energy is dissipated. As far as we are aware, the only independent measurements of energy dissipation and mixing in unsteady breaking waves are those of RM. In this paper we shall show that taken together, modeling by Phillips (1985) of the dissipation due to breaking and RM's laboratory measurements of dissipation and mixing imply the existence of a layer of enhanced dissipation consistent with the measurements of Agrawal et al. (1992).

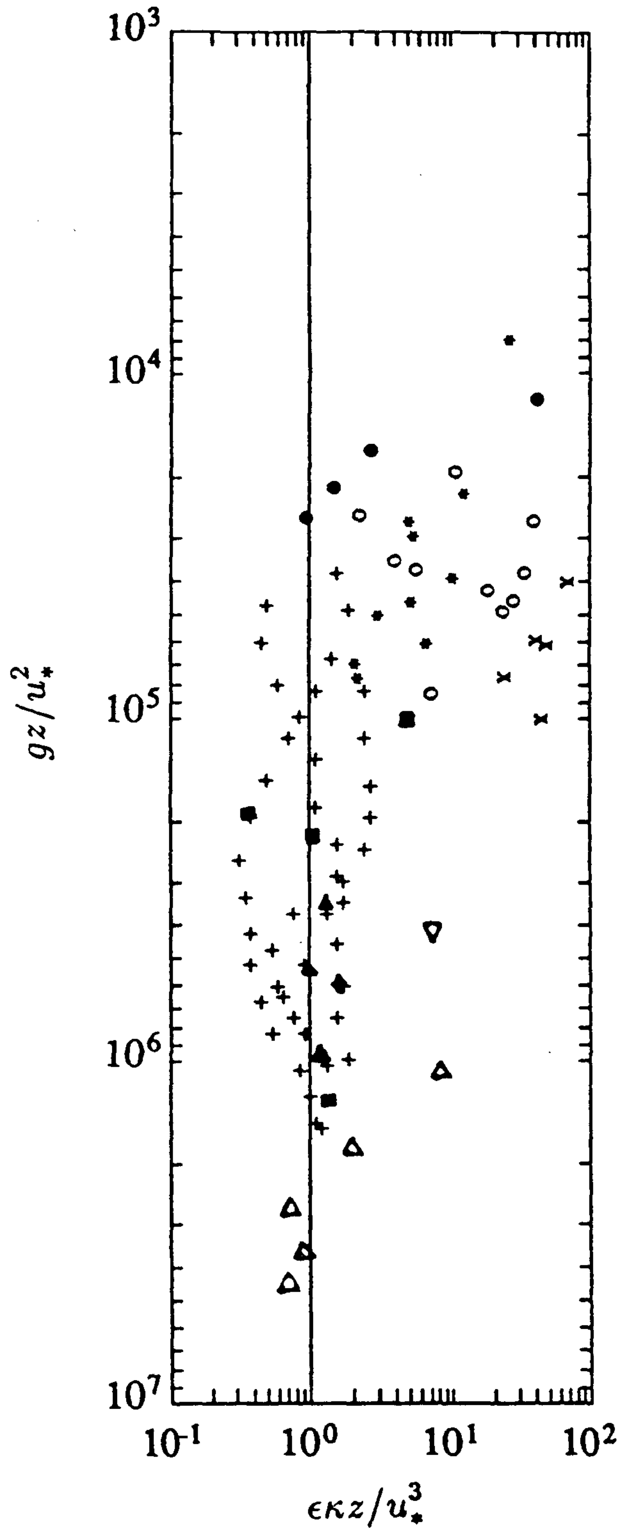

FIG. 1. Dissipation in "wall-layer" coordinates $\epsilon\kappa z/u_{*w}^3$ vs zg/u_{*w}^2, measured (○, $*$, ●) and collated (□, +, △, ◇, ×, △, ▽) by Agrawal et al. (1992). Collated data cited by Jones (1985) and Soloviev et al. (1988) include lake and ocean data using fixed and moving sensors. The vertical line represents the dissipation level in a conventional boundary layer over a rigid surface.

During the course of this work it was learned that Thorpe (1993) was also investigating dissipation in the surface layers. Whereas Phillips (1985) used his estimates of the total dissipation and Duncan's (1981) measurements of the rate of dissipation to infer the breaking statistics, Thorpe used measured breaking statistics and Duncan's measurements to estimate the total dissipation. He compared this estimate with measurements of dissipation in the lower half of the mixed layer. If the ideas proposed here are correct—that is, that essentially all of the dissipation due to breaking is contained in a surface layer of thickness comparable to the wave height—then estimates of energy fluxes across the interface based on measurements deeper in the mixed layer may need revision (Thorpe 1993). However, use of Duncan's (1981) estimates of dissipation rates by both Phillips and Thorpe implicitly assumes that results obtained for quasi-steady breaking apply to the unsteady breaking observed in the field. We discuss this assumption in the light of recent laboratory measurements of unsteady breaking by Loewen and Melville (1991), which lead to estimates of dissipation rates up to an order of magnitude less than those measured in quasi-steady breaking waves.

2. Predictions of dissipation by Phillips

Phillips (1985) considered an equilibrium range for wavenumbers large compared with the spectral peak. By assuming a balance between wind input, nonlinear transfers, and dissipation by breaking, and by arguing that the equilibrium range has no internal wavenumber scale, he was able to show that the three terms are proportional. Using theoretical and empirical results to model the wind input and nonlinear transfers he obtained the following form for the wavenumber spectrum:

$$\psi(\mathbf{k}) = \beta(\cos\theta)^p u_{*a} g^{-1/2} k^{-7/2}, \tag{1}$$

where β and p are numerical constants, θ is the included angle between the wavenumber vector $\mathbf{k}$ and the wind vector, and u_{*a} is the friction velocity in the air. The spectral rate of energy loss from the wave components in the equilibrium range was given by

$$\epsilon(\mathbf{k}) = \gamma\beta^3(\cos\theta)^{3p} u_{*a}^3 k^{-2}, \tag{2}$$

where γ is a numerical constant. Phillips pointed out that this result implies that the energy source for the near-surface turbulence due to breaking is, in the mean, distributed over a wide range of scales since the distribution over scalar wavenumber k is proportional to $u_{*a}k^{-1}$, while the larger scales of the turbulence are at least proportional to (if not comparable to) the wavelength.

If the equilibrium wavenumbers are in the range (k_0, k_1), then the total dissipation due to wavenumbers in the equilibrium range is

$$\epsilon_0 = 2\int_{-\pi/2}^{\pi/2}\int_{k_0}^{k_1} \epsilon(\mathbf{k}) k dk d\theta = 2\gamma\beta^3 I(3p) u_{*a}^3 \ln\frac{k_1}{k_0}. \tag{3}$$

Assuming k_0 could be approximated by the wavenumber at the peak of the spectrum, and that $k_1 = rg/u_{*a}^2$, where r is of order unity, Phillips approximated Eq. (3) by

$$\epsilon_0 \approx \left(2\gamma\beta^3 I(3p)\frac{\rho_w}{\rho_a}\right)\rho_a u_{*a}^3 \ln\left[r\left(\frac{c_0}{u_{*a}}\right)^2\right], \tag{4}$$

where c_0 is the phase speed at the spectral peak. The last factor can be rewritten as

$$\ln\left[\frac{r}{C_D}\left(\frac{c_0}{U}\right)^2\right],$$

where U is a reference wind velocity (usually the neutral wind at 10 m), and C_D is the aerodynamic drag coefficient based on the reference wind. Thus,

$$\epsilon_0 \approx A \ln\left[\frac{r}{C_D}\left(\frac{c_0}{U}\right)^2\right]\rho_a u_{*a}^3, \tag{5}$$

where A represents the numerical factor in parentheses in Eq. (4). Recall that this is a conservative estimate since it only accounts for the dissipation in the equilibrium range.

3. The measurements of Rapp and Melville

Rapp and Melville (1990) undertook an extensive series of laboratory experiments on unsteady breaking using flow visualization, wave gauge measurements, and laser anemometry to study the kinematics and dynamics of breaking. Using wave gauge measurements upstream and downstream to measure the energy dissipation, and laser anemometer measurements of the velocity field in the turbulent fluid directly mixed down by breaking, RM concluded that more than 90% of the total energy lost from the wave field was dissipated within four wave periods after the inception of breaking. The active breaking itself lasted for a time comparable to the wave period. This very rapid rate of dissipation was difficult to reconcile with our preconceptions until more recent experiments (Lamarre and Melville 1991) showed that up to 50% of the energy lost from the wave field was expended in entraining air against the effect of buoyancy forces. Rapp and Melville also showed that the turbulent region generated by the breaking wave mixed down to a depth D, with $kD \approx 0.5 - 1$ after four wave periods. Here k is a characteristic wavenumber. These data are shown in Fig. 2. It can be seen that the initial deepening of the layer is very rapid during the first wave period after breaking, subsequently reaching an asymptotic dependence $D \propto t^{1/4}$ after one to two wave periods. Since essentially all of the dissipation must take place within

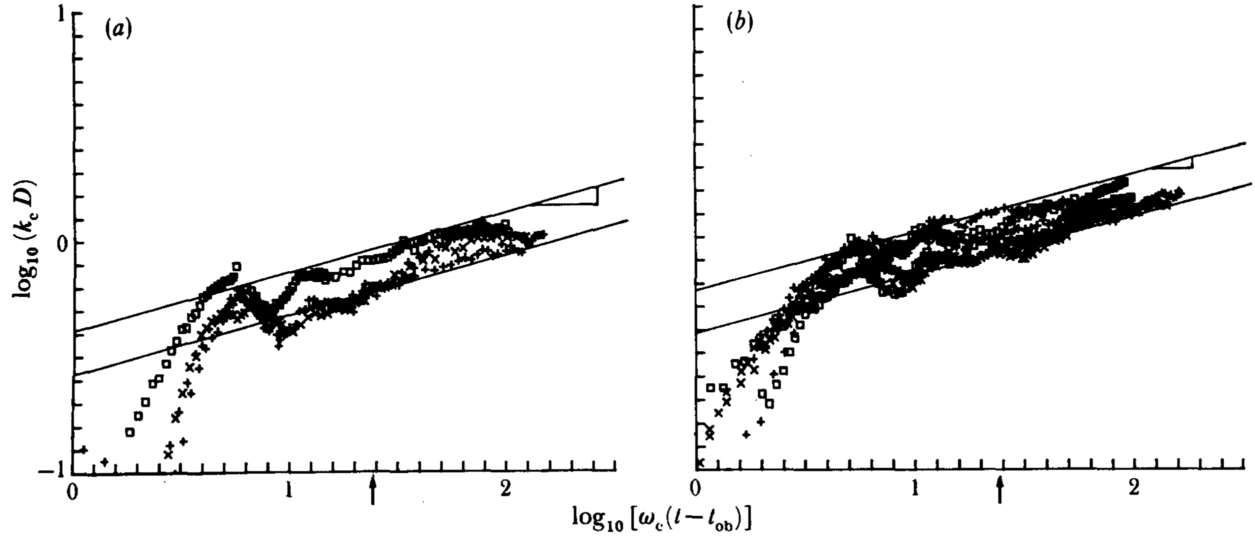

FIG. 2. Measurements by Rapp and Melville (1990) of the normalized depth, $k_c D$, of the fluid mixed down by unsteady breaking waves plotted as a function of normalized time $\omega_c(t - t_{0b})$, where k_c and ω_c are characteristic wavenumbers and frequencies, respectively, and $t - t_{0b}$ is the time from the beginning of breaking: (a) spilling breakers; (b) plunging breakers. The different symbols refer to different runs at different frequencies and wavenumbers. Note the initial rapid deepening followed by an asymptotic regime in which $D \propto t^{1/4}$. The arrows mark the nominal four wave periods after breaking, by which time RM measured more than 90% of the wave energy lost by breaking to be dissipated in the water column.

this layer, this implies that the bulk of the dissipation due to breaking will take place in a layer whose depth is much less than the wavelength. Laboratory experiments (Bonmarin 1989; RM) show that breaking occurs for wave slopes ak in the range 0.2–0.3. Thus, after four wave periods this highly dissipative layer would be in the range of 1–2 wave heights thick. Subsequently, even after O(100) wave periods, the depth of this layer is still of the order of the wave height.

4. The dissipative surface layer

The Phillips predictions of the dissipation due to breaking strictly apply only to the equilibrium layer, which is believed to begin at wavenumbers somewhat larger than the spectral peak. However, since the integrated dissipation only depends logarithmically on this lower limit the final result is not sensitive to its particular value. Can we assume that the equilibrium range provides the bulk of the dissipation? The answer to this question is not clear. We expect that there will be dissipation near the peak of the spectrum, which is balanced to some extent by nonlinear transfers from higher wavenumbers (Komen et al. 1984). What we can conclude is that the total dissipation will not be smaller than that contributed by the equilibrium range. To be conservative, let us assume that all dissipation is given by the Phillips model.

We assume that the energy lost from the wave field is dissipated over a depth D at a volumetric rate ϵ_w,

$$\epsilon_0 = \int_0^D \rho_w \epsilon_w dz = \rho_w \bar{\epsilon}_w D, \tag{6}$$

which defines the mean dissipation $\bar{\epsilon}_w$ over the depth D. Given the large initial rates of mixing and dissipation following breaking found in the laboratory, we believe that the depth of the dissipative layer in the ocean will be of the same order of magnitude as that found for single breaking events. More specifically, we expect that the mixing and dissipation are highly intermittent and dominated by the individual breaking events, with the background levels of turbulence being considerably lower. Thus, we use D to represent both the depth of mixing due to a single event and the depth of the dissipative layer. Using $\bar{\epsilon}_w$ and D to scale ϵ_w and z, respectively, it follows that

$$\frac{\epsilon_w \kappa z}{u_{*w}^3} = \mathrm{O}\left(\frac{\bar{\epsilon}_w \kappa D}{u_{*w}^3}\right) = \frac{\epsilon_0 \kappa}{\rho_w u_{*w}^3}$$
$$> \kappa A \ln\left[\frac{r}{C_D}\left(\frac{c_0}{U}\right)^2\right] \rho_a u_{*a}^3 \Big/ \rho_w u_{*w}^3. \tag{7}$$

The momentum flux lost from the wave field due to breaking will be transferred to currents. Given the intense mixing due to breaking, we anticipate that there will be a distinct surface current within a depth of the order of one wave height of the surface [i.e., O(D)]. Since we are assuming the wave field is in approximate local equilibrium, to be consistent we must also assume that horizontal gradients in the current are not large, and in consequence are changing on the same slow spatial scales as the wave variables. This implies that the momentum flux across the upper boundary of the current (the surface) is almost matched by that across its lower boundary at a depth comparable to the wave height. Thus, we anticipate that $\rho_a u_{*a}^2 \approx \rho_w u_{*w}^2$, whence $u_{*a}/u_{*w} \approx (\rho_w/\rho_a)^{1/2}$, and

$$\frac{\bar{\epsilon}_w \kappa D}{u_{*w}^3} > \left(\frac{\rho_w}{\rho_a}\right)^{1/2} \kappa A \ln\left[\frac{r}{C_D}\left(\frac{c_0}{U}\right)^2\right]. \quad (8)$$

A preliminary examination of this expression shows that it is dominated by $(\rho_w/\rho_a)^{1/2} \approx 28$. The factor A, based on the Phillips (1985) estimates of its constituent factors, is estimated to be >1.3 for $r = 0.16$, or >0.8 for $r = 0.5$. The logarithmic factor will depend on the wind speed and fetch. For a fixed wind speed, the effect of the increase in wave age,[1] c_0/U, with fetch is stronger than the decrease in C_D and the logarithmic factor increases, consistent with an increase in the bandwidth of the equilibrium range. For a fixed fetch, the effect of the decrease in wave age is stronger than the increase in C_D, and the logarithmic factor decreases. Estimated minimum values of the normalized dissipation rate for three wind speeds and two fetches are presented in Table 1. The values in the table are based on the Phillips estimates of the parameters appearing in his theory (for a spreading parameter, $p = 0.5$); the data of Donelan et al. (1992) for the wave age as a function of fetch, and the data of Maat et al. (1991) for the drag coefficient, C_D. The high wavenumber cutoff of the equilibrium range is proportional to r (see above). The 1.1 km fetch and the lowest wind speed of 8 m s^{-1} were chosen to correspond to the lake data of Agrawal et al. (1992). The longer fetch was chosen to be typical of many oceanic conditions. At greater fetches, say 1000 km, the wind sea would be fully developed at these wind speeds and the logarithmic term in inequality (8) would reduce to $\ln(r/C_D)$, which with a drag coefficient of 0.002 would give estimates of the minimum dimensionless dissipation rate of 160 and 122 for $r = 0.16$ and 0.5, respectively. These values are essentially the same as those for 8 m s^{-1} wind at 100 km fetch. To summarize these results, they show that, except at small fetches with high winds, the dissipation rate will be one to two orders of magnitude greater than that in a comparable wall layer.

It is of interest to note from Fig. 1 that for $\tilde{z} = gz/u_{*w}^2 < 10^5$ the normalized dissipation is up to 70 times greater than that for the conventional wall region. This compares favorably with our conservative estimate of an enhancement by a factor of 60 or more at the fetch of the measurements of Agrawal et al. (1992). With the exception of two points, for $\tilde{z} > 10^5$ the normalized dissipation is of O(1), consistent with the wall scaling.

[1] The wave age is sometimes defined to be c_0/u_{*a}.

TABLE 1. Estimated minima of normalized dissipation rates as a function of wind speed and fetch, based on estimates of Phillips (1985) and fetch-dependent data of Donelan et al. (1992) and Maat et al. (1991). The high wavenumber limit of the equilibrium range is proportional to r.

Wind speed (m s^{-1})	Fetch = 1.1 km		Fetch = 100 km	
	$r = 0.16$	$r = 0.5$	$r = 0.16$	$r = 0.5$
8	69	66	160	118
10	39	49	143	111
15	4	27	113	93

Unfortunately, Agrawal et al. (1992) did not give complete supporting data for their measurements; however, they did indicate that the fetch-limited waves had significant wave heights, H_s, of the order of 30 cm, and the winds were greater than 8 m s^{-1}. Taking these values and assuming a value of the drag coefficient of 2×10^{-3}, the scaled significant wave height, $\tilde{H}_s = 1.8 \times 10^4$. From our arguments above we expect the layer of enhanced dissipation to be in the range of 1–2 wave heights [i.e., O(D)], giving a dimensionless thickness in the range 2–4 ($\times 10^4$). This estimate is entirely consistent with the Lake Ontario data of Fig. 1, and the dissipation layer having a dimensionless thickness of O($10^4 - 10^5$). Taking this agreement as support for the thickness of the layer being 1–2 wave heights, we can use the empirical correlations of the dimensionless variance of the surface displacement with wave age (Donelan et al. 1992) to conjecture that the depth of the dissipative layer, D, is given by

$$\frac{Dg}{U^2} = \alpha\left[\frac{U}{c_0}\right]^{-1.6}, \quad (9)$$

where α is in the range 0.2–0.4, U is the equivalent neutral wind at 10 m, and c_0 is the phase speed at the peak of the spectrum.

5. Rates of dissipation in breaking waves

The estimates of energy lost from the wave field by breaking described above are dependent only on wind–wave modeling. However, there is another approach, also formulated by Phillips (1985), in which laboratory measurements by Duncan (1981) of the dissipation rate in quasi-steady breaking waves and the dissipation predictions based on the wind–wave modeling were used to infer the statistics of wave breaking. Most recently Thorpe (1993) has used measured breaking statistics and Duncan's results to infer the dissipation. The issue here is the relevance of measurements of quasi-steady breaking to models of unsteady breaking in the field.

Breaking ship wakes may be quasi-steady. Small microscale breaking waves that are strongly forced by the wind appear to be quasi-steady, propagating for several wavelengths without obvious change of form; but breaking of sufficient scale to lead to whitecapping appears to be unsteady, lasting for some fraction of a wave period. However, it is very difficult to assess from casual observations whether breaking would be quasi-steady in a suitable frame of reference. It is better to rely on quantitative observations. At present the only available quantitative observations of the details of unsteady breaking are those made in the laboratory. Both Bonmarin (1989) and RM have measured the evolution of breaking waves in the laboratory. Do their measurements show that such breaking is essentially unsteady, or is it quasi-steady in a suitable reference frame? Bonmarin's measurements of breaking following hydrodynamic instability show the wavelength decreasing by approximately 20%, the wave height decreasing by approximately 50%, and the steepness of the forward face of the wave decreasing by approximately 75% *during breaking* (Bonmarin 1989, Figs. 6, 7, 10, respectively). Rapp and Melville were less concerned with the evolution of the geometry of the wave, but their measurements of breaking due to constructive interference also imply that the breaking is intrinsically unsteady. They show that the time to establish a spilling whitecap is a large fraction of the total duration of the event (RM, Fig. 8). Further, using the data of RM, Melville and Rapp (1988) have shown that the turbulence generated by unsteady breaking in the laboratory does not scale as wake turbulence, which is the case for quasi-steady breaking. We conclude from this evidence that much of the breaking in the field, especially at scales leading to whitecapping, is likely to be intrinsically unsteady.

Duncan (1981) measured quasi-steady breaking waves generated by a hydrofoil towed at constant speed. His measurements of the drag of the whitecap per unit width on the underlying flow, F_b, was given by[2]

$$F_b = \frac{0.009\rho_w C^4}{g \sin\theta}, \tag{10}$$

where C is the phase speed of the wave, and θ is the angle of inclination of the breaking region to the horizontal. Since the speed of the underlying fluid is approximately C, the dissipation per unit length of breaking crest is approximately $0.009\rho_w C^5/g \sin\theta$. Now Duncan's tabulated results show that the inclination of the breaking region over his twelve experiments was in the range 12.5 ± 2.5 deg, whence the dissipation rate per unit length, ϵ_l, was given by

$$\epsilon_l = (0.044 \pm 0.008)\frac{\rho_w C^5}{g}. \tag{11}$$

Phillips (1985) quotes Duncan's (1981) result divided by ρ_w and uses a slightly higher numerical factor of 0.06. The use of this result, specifically, the assumption that the numerical factor is constant, implies that all breaking waves with a characteristic phase velocity C will give the same rate of dissipation. Duncan's (1981) tabulated results show that the inclination of the forward face of the wave is constant to within ± 2.5 deg, *or* $\pm 20\%$ over the range of his experiments. Based on the stated accuracy of the distance measurements of ± 0.2 cm and lengths of the breaking regions in the range 7–21 cm, the inclination of the breaking region would have been measured to within ± 1 deg at best, and ± 3 deg at worst. Given this accuracy and the measurements, the assumption of a constant coefficient appears justified. However, subsequent measurements by Duncan (1983) show significant changes in the inclination of the forward face of the breaking wave with changes in the depth of submergence of the hydrofoil. Duncan's (1983, Figs. 6, 8) later results imply that

$$\epsilon_l = 0.0075\frac{\rho_w C^5}{g \sin\theta}, \tag{12}$$

with θ in the range 6.5–14 deg, giving values of $\epsilon_l g/\rho_w C^5$ in the range 0.066–0.031. Thus, Duncan's (1983) results clearly show a change in the dimensionless dissipation rate with a change in his control parameter. Nevertheless, Duncan's (1981, 1983) measurements were of quasi-steady breaking waves.

Surface waves, whether breaking or not, are characterized by an amplitude as well as a phase speed and dispersion relationship. From dimensional reasons alone then we would expect that the rate of dissipation would not depend only on the phase speed of the wave. For example, we would expect that a spilling wave may have a lower dissipation rate than a more vigorous plunging wave. Melville and Rapp (1986) and RM have shown that the total energy dissipated or "strength" of breaking may be quantified by an integral slope parameter. As far as we are aware there are no published results on the *rate* of dissipation due to unsteady breaking; however, the published results of Loewen and Melville (1991, hereinafter LM) may be used to determine a dissipation rate.

The measurements of LM were not designed to address this issue. They were concerned with the acoustics of, and microwave scattering by, breaking waves. Loewen and Melville generated breaking waves in a manner similar to that of RM with the exception that the component waves in the packet were of constant slope rather than constant amplitude. Loewen and Melville measured the energy lost from the wave field and the duration of the sound generated by the breaking waves. Observations showed that the sound began at the time the crest struck the surface below and continued so long as there was active entrainment of air at the front of the break. The sound is believed to be generated by the volumetric oscillations of the entrained bubbles as they relax back to their equilibrium

[2] A typographical error in Duncan's Eq. (17) led to the omission of the fluid density ρ_w.

spherical shape. Rapp and Melville and LM have found that beyond the use of characteristic wavenumbers, frequencies, or phase speeds for normalizing the data, an integral measure of the slope of the wave packet has proven to be most useful in correlating data of different characteristic wavenumbers. In this case we use the slope $S = Nak$ where N is the number of discrete wave components in the packet and ak is the slope of each component. Thus, S is the maximum slope that linear superposition of the components would yield.

We have taken the measurements of energy dissipated and the duration of the whitecap and normalized the values by $\rho_w C^5/g$ to give dimensionless rates of dissipation, which may be compared with the numerical values of 0.044 ± 0.008 derived from Duncan (1981) and 0.031–0.066 derived from Duncan (1983). The results are shown plotted against S in Fig. 3 for three separate wave packets having center component phase speeds of 1.55, 1.38, and 1.20 m s^{-1}, and frequencies of 0.88, 1.08, and 1.28 Hz, respectively. In these experiments, single breaking waves occurred for $S < 0.3$, whereas multiple breaking events occurred for $S \geqslant 0.3$. In the latter case we have simply summed together the durations of the multiple events since we have no way of determining the dissipation due to each event individually.

Before discussing the results it should be noted that the duration data for the highest frequency packet is the least reliable due to a low signal-to-noise ratio, but is included here for completeness. These waves generated the least noise and the duration may be biased toward lower values since the signal-to-noise ratio was lower. The data for the two higher frequencies had a significantly higher signal-to-noise ratio and it can be seen that it collapses onto a single curve for $S < 0.3$, with the dimensionless rate of dissipation increasing monotonically with S. There is a break at $S \approx 0.3$, corresponding to the onset of multiple events, and thereafter the rate of dissipation is approximately constant within the scatter of the results. That the dissipation rate for the multiple events is at an intermediate value is consistent with the observation that multiple events were never comprised of all weak events (spilling breakers) nor all strong events (plunging breakers). The data for the highest frequency packet show the same qualitative behavior, but for $S < 0.3$ the values are higher by a factor in the range 1.3–2. This is most likely due to the lower signal-to-noise ratio in this case leading to an underestimate of the duration of the break. In addition to the dependence on S, which shows that the dimensionless rate of dissipation increases as the waves progress from spilling to plunging, the dimensionless dissipation for the gently spilling waves is up to an order of magnitude less than that measured in the quasi-steady breaking waves of all strengths. This difference may have a profound effect on the reliability of quantitative conclusions based on the quasi-steady estimate.

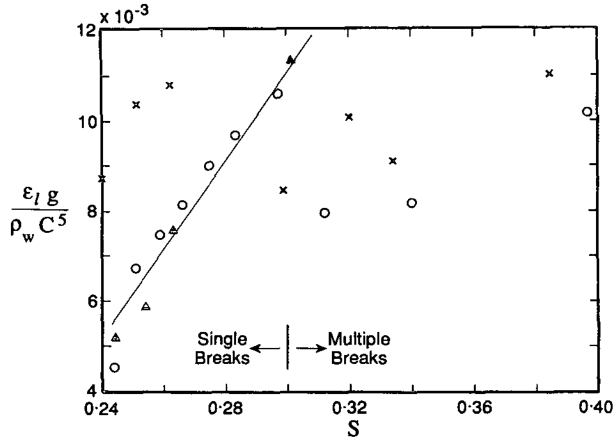

FIG. 3. Normalized dissipation rate per unit length of crest in unsteady breaking waves, $\epsilon_l g/\rho_w C^5$, vs integral wave slope S, inferred from total dissipation and breaking duration measurements of Loewen and Melville (1991). Characteristic frequencies are 0.88 Hz: △, 1.08 Hz: ○, and 1.28 Hz: ×. The data at the highest frequency, or for multiple breaks, are believed to be least significant. The straight line displays the trend of the most significant data.

In view of this apparent quantitative discrepancy between the rates of dissipation in gently breaking, or spilling, quasi-steady and unsteady breaking waves, it is worth trying to make an independent estimate of the dissipation rate in unsteady breakers. We assume that the energy is ultimately lost through viscosity to heat. For turbulent flows of sufficiently large Reynolds numbers, the rate of dissipation per unit mass is typically $O(\tilde{u}^3/l)$, where $\tilde{u}$ is an integral velocity scale, and l is an integral length scale. Rapp and Melville's dye measurements showed that the roughly triangular turbulent region extended to a depth D and a length comparable to the wavelength λ. Thus the dissipation rate per unit length of crest, ϵ_l is given by

$$\epsilon_l \approx \frac{\rho_w \tilde{u}^3}{l}\frac{D\lambda}{2}. \tag{13}$$

Now RM found that all the lengths and velocities scaled with the wave variables. Thus, we anticipate that $\tilde{u} = \chi C$, where χ is a numerical constant. We also expect that the largest eddies are comparable in length scale to the depth of the turbulent patch, D. Now $\lambda = 2\pi C^2/g$, whence

$$\epsilon_l \approx \frac{\rho_w \pi}{g}(\chi C)^3 C^2. \tag{14}$$

We now have to estimate χ. Rapp and Melville found that the initial deepening of the turbulent patch was such that $kD = 0.3$, 0.5 within half a wave period for spilling and plunging waves, respectively. This corresponds to vertical velocities of $0.1C$ and $0.17C$, respectively. Approximately four wave periods after breaking the rms turbulent velocities had reduced to approximately $0.02C$. Given RM's finding that more than 90% of the dissipation occurred within four wave periods of breaking, we expect that estimates of the turbulent velocity scales should be weighted more to the initial values than the lower later values. Accordingly, we estimate χ to be in the range 0.1–0.17. Substituting these values for χ in Eq. (14), we get

$$\epsilon_l \approx (3.2 \times 10^{-3}, 1.6 \times 10^{-2}) \frac{\rho_w C^5}{g}, \quad \chi = 0.1, 0.17. \tag{15}$$

Given the fact that these are order of magnitude estimates, these numerical coefficients compare quite favorably with the range 4–12 ($\times 10^{-3}$) shown in Fig. 3, based on the data of LM. Recall that the lower values correspond to spilling whitecaps, which are likely to be prevalent in the field. While plunging breakers can occur in deep water, we expect them to occur less frequently.

6. Discussion

We have shown that the Phillips (1985) prediction of the energy lost from the wave field by breaking, along with the laboratory measurements of dissipation and mixing by RM, lead to the conclusion that there is a layer of enhanced dissipation at the surface having a thickness of the order of the wave height. This conclusion is completely consistent with the recent measurements by Agrawal et al. (1992) of enhanced dissipation in the surface layer.

Thorpe (1993) has used measurements of the incidence of breaking as a function of wave age, and Duncan's (1981) data [Eq. (11)] to estimate the rate of energy loss from the waves, E_w, where

$$E_w = (3.0 \pm 1.8) \times 10^{-5} \rho_w U^3 \left(\frac{c_b}{c_0}\right)^5, \tag{16}$$

and c_b is the characteristic phase speed of the breaking waves. Comparing this expression with estimates of dissipation based on measurements in the lower half of the mixed layer (Oakey and Elliot 1982), it was concluded that if the energy lost from breaking were to support the turbulence in the mixed layer, $c_b/c_0 = 0.25$. If c_b/c_0 were unity the energy flux from the breaking waves would have been 1000 times too large! If, however, we use the estimates of dissipation rates for spilling unsteady waves (cf. Fig. 3), which are an order of magnitude smaller than those of Duncan (1981, 1983), and if we accept that at fetches of O(100 km), comparable to those of Oakey and Elliot (1982), there is a surface layer an order of magnitude shallower than the mixed layer in which the dissipation rates are one to two orders of magnitude higher (cf. Table 1), then $(c_b/c_0)^5 = O(0.01 - 0.1)$, and $c_b/c_0 \approx 0.40 - 0.63$. These estimates are consistent with a numerical evaluation of Eq. (5) based on the entries in Table 1 for 100-km fetch, which give

$$\epsilon_0 \approx (4.6 \pm 1.6) \times 10^{-7} \rho_w U^3. \tag{17}$$

Equating this estimate of ϵ_0 with Thorpe's E_w, reduced by a factor of 7 to account for the decreased dissipation in unsteady spilling waves (cf. Fig. 3), gives $c_b/c_0 = 0.64$. It should be emphasized that implicit in estimates of energy fluxes based on the equilibrium region is that these waves have smaller phase speeds than those at the peak of the spectrum. Quite correctly, Thorpe (1993) has pointed out the implication of these phase speed ratios for length scales, since these are proportional to the square of the phase speed. In this regard, $c_b/c_0 = 0.25$ gives a wavelength ratio of 0.06, whereas $c_b/c_0 = 0.40, 0.64$ gives a length ratio of 0.16 to 0.41: perhaps up to an order of magnitude different.

Enhanced dissipation at the surface has other than dynamical interest for processes of air–sea interaction. Recent field measurements by Wallace and Wirick (1992) have shown evidence of significant increases in air–sea gas fluxes that were associated with increases in surface wave activity, and indirectly with breaking. Kitaigorodskii (1984) has modeled the influence of patches of enhanced turbulence due to breaking on gas transfer. He found that the transfer velocity was proportional to $\mathrm{Sc}^{-1/2}[\nu\epsilon_\nu(0)]^{1/4}$, where Sc is the Schmidt number, the ratio of the kinematic viscosity of the fluid to the diffusion coefficient of the dissolved gas, and $\epsilon_\nu(0)$ is the dissipation rate at the surface. If Kitaigorodskii's model is correct, we would expect an enhancement of the dissipation by a factor of 60 to lead to an increase in the transfer velocity by a factor of 3. Jaehne (1990) has found that the addition of a mechanically generated breaking wave to a wind-generated breaking wave in the laboratory can enhance gas transfer by a factor of 2.

The formulation by Phillips (1985) of the dynamics of the equilibrium range, the fluxes of momentum and energy from the wave field to the water column, and the statistical description of the breaking waves provides a general method of approach extending beyond the quantitative details of his results. When combined with independent estimates of the rate of dissipation due to breaking, the Phillips formulation provides a rational approach to predictions of many of the quantities of interest relating to breaking and its influence on air–sea fluxes. However, the available laboratory data on dissipation rates in breaking waves show differences of up to an order of magnitude between quasi-steady and unsteady breaking waves, and differences of up to an order of magnitude between spilling and plunging unsteady breakers. Plausible independent estimates of dissipation rates in unsteady breaking waves based on the kinematic measurements of RM give good agreement with the rates inferred by LM's data. However, this agreement should be regarded with some caution, depending as it does on the cube of estimated parameters! Given the importance of processes of air–sea interaction in meteorology and oceanography, it is imperative that the uncertainties that exist be resolved.

Acknowledgments. I wish to thank Gene Terray and Mark Donelan for preprints of Agrawal et al. (1992), Mark Loewen and Jim Duncan for fruitful discussions of the laboratory measurements of dissipation, and Hans Graber for discussions of drag coefficients. I am grateful to Steve Thorpe for stimulating discussions on the mixed layer, which prompted me to write up this work. The essential ideas described here were presented in August 1992 at the 18th International Congress of Theoretical and Applied Mechanics, Haifa, Israel (Melville 1992). This work is supported by NSF Grant OCE91-15594 and by the Office of Naval Research.

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